GODDARD GRANT
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RF Project 761054/711055
Report No. 375

DETERMINATION OF EARTH ROTATION BY THE COMBINATION OF DATA FROM DIFFERENT SPACE GEODETIC SYSTEMS

Brent Allen Archinal
Department of Geodetic Science and Surveying

(NASA-CR-181388) DETERMINATION OF EARTH ROTATION BY THE COMBINATION OF DATA FROM DIFFERENT SPACE GEODETIC SYSTEMS (Ohio State Univ.) 104 p CSCL 08E

N88-20713

Unclas
G1/43 0134773

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Goddard Space Flight Center
Greenbelt, Maryland 20771

Grant No. NSG 5-265, Supp. No. 10

February 1987

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The Ohio State University
Research Foundation
1314 Kinnear Road
Columbus, Ohio 43212

DETERMINATION OF EARTH ROTATION BY THE COMBINATION OF DATA FROM DIFFERENT SPACE GEODETIC SYSTEMS

by

Brent Allen Archinal

Report No. 375

Department of Geodetic Science and Surveying
The Ohio State University
Columbus, Ohio 43210-1247

February, 1987

DEDICATION

to JoAnne
. . . who brought clear
skies to my life . . .

PREFACE

This project is under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science and Surveying, The Ohio State University. The science advisor is Dr. David E. Smith, and the technical officer is Dr. Gilbert B. Mead, both at Code 601, Crustal Dynamics Project, Space and Earth Sciences Directorate, NASA Goddard Space Flight Center, Greenbelt, Maryland 20771. The work is carried out under NASA Grant No. NSG 5265, OSU Research Foundation Project No. 711055.

This report was originally presented as a dissertation to the Graduate School of The Ohio State University in partial fulfillment of the requirements for the PhD degree.

ABSTRACT

In the past Earth Rotation Parameters (ERP), i.e., polar motion and UT1-UTC values, have been determined using data from only one observational system at a time, or by the combination of parameters previously obtained in such determinations. The question arises as to whether a simultaneous solution using data from several sources would provide an improved determination of such parameters. In order to consider the promise of this more fully, 15 days of observations have been simulated using realistic networks of Lunar Laser Ranging (LLR), Satellite Laser Ranging (SLR) to Lageos, and Very Long Baseline Interferometry (VLBI) stations. Then a comparison has been done of the accuracy and precision of the ERP obtained from: a) the individual system solutions, b) the weighted means of those values, c) all of the data, via the combination of the normal equations obtained in a), and d) a grand solution with all the data.

These simulations show that solutions done by the normal equation combination and grand solution methods provide the best or nearly the best ERP for all of the periods considered, but that weighted mean solutions provide nearly the same accuracy and precision. VLBI solutions also provide ERP of similar accuracies. The simulations also indicate that ERP recovery at the 1 mas level for polar motion, and the 0.2 to 1.0 ms level for UT1-UTC is at the limit of all current techniques, without increasing the observational accuracies, or the number of stations.

ACKNOWLEDGMENTS

Many people and organizations have contributed their help and advice in my undertaking of the work for this study. Although it is difficult to adequately do so here, some acknowledgment of their help must be made and thanks given.

First and foremost, my deepest thanks and appreciation goes to my wife JoAnne and my parents, for the long and continued patience they have shown during all of my work here. Without their continued faith, this work would almost certainly not have been completed.

Second, I would like to thank my adviser, Dr. Ivan I. Mueller, for providing me the opportunity to work with him during my graduate work here at Ohio State. His advice, encouragement, and constructive criticism have all been greatly appreciated. His continued patience and assistance in obtaining financial support has been especially appreciated, during the sometimes seemingly endless amount of time spent on this and earlier work.

Thanks are also due to Dr. Richard H. Rapp and Dr. Urho A. Uotila, not only for their time in reading this report and providing suggestions for its improvement, but also for their overall guidance during my coursework with them here at Ohio State.

Acknowledgement is also made of the help provided by the many students of the Department of Geodetic Science and Surveying. The substantial help and camaraderie provided by the students has always been an exceptional feature of this Department during my time here. More specifically, the valuable discussions with George Dedes, not only on this study, but also on other topics has been greatly appreciated. The support of former students such as Dr. Erricos C. Pavlis (now of EG&G-Washington Analytical Science Center (WASC)) and Dr. Lenny Krieg in helping to learn to use the computer facilities here is also much appreciated. This help has proven quite invaluable considering the extensive computer work required in undertaking this study. The advice of Dr. Yehuda Bock (now of Massachusetts Institute of Technology/Air Force Geophysics Laboratory (MIT/AFGL)), both before and after he received his degree here, has been most helpful, both in understanding some of the principles of VLBI observations, and in his assistance in using his VIP program. Information on LLR from Dr. John Luck (now of the Department of National Mapping (NATMAP), Australia) is also appreciated.

As much dependent as this study eventually became on the use of the NASA GEODYN and SOLVE programs, special mention must be made of the excellent help and assistance provided by personnel of EG&G-WASC and NASA

Goddard Space Flight Center (GSFC). At WASC, special thanks is given to Dr. Nikita Zelensky for his substantial help in getting a version of GEODYN operating here capable of processing VLBI data, and in answering many questions on VLBI data processing and the use of the SOLVE program. Dr. Pavlis, Dr. Peter Dunn, David Rowlands, Mark Torrence, and John Robbins are all to be thanked for their assistance in using the GEODYN program. Ms. Barbara Putney of GSFC provided or assisted in supplying the many versions of GEODYN and SOLVE that were installed here. Dr. Chopo Ma of GSFC also helped in resolving several questions on VLBI data processing.

Since there were three very complex observational systems under consideration in this study, the advice and time provided by several people in these areas was invaluable in trying to understand the details of the various systems. In the area of LLR, the advice of Dr. Peter Shelus of the University of Texas (UTX) Dept. of Astronomy, and of Dr. Robert King of MIT was invaluable. Likewise, in the field of SLR, discussions with Dr. Bob Schutz and Richard Eanes of the UTX Dept. of Aerospace Engineering were very helpful. In the area of VLBI much advice and assistance was provided by Dr. William E. Carter, Dr. Douglas Robertson, Dr. Frederick W. Fallon, and J. Ross MacKay, all of the National Geodetic Survey (NGS). Some additional information on VLBI was also provided by Dr. Nancy Vandenberg of Interferometrics, Inc. Also greatly appreciated was information provided by Dr. Dennis McCarthy of the U.S. Naval Observatory, which dealt with several topics, such as LLR, Connected Element Radio Interferometry (CERI), and ERP combination solutions.

A special acknowledgement is also made to Dr. Shelus and Dr. Carter for their help in arranging visits to McDonald Observatory and the George R. Agassiz Station (GRAS) respectively, near Ft. Davis, Texas.

Thanks also go to GSFC/NASA for providing funds for its continuing and important support of basic research in geodynamics, including the financial support of this project through NASA Research Grant No. NSG 5265, OSURF Project No. 711055. The financial support provided by the Dept. of Geodetic Science and Surveying as a Graduate Teaching Associate is greatly appreciated, as is the substantial computer funds it provided for use at the Instruction and Research Computer Center of The Ohio State University. Acknowledged also is the financial support of the Wild Heerbrugg - American Congress on Surveying and Mapping (ACSM) Geodetic Fellowship (1983).

This report was originally presented as a dissertation to the Graduate School of The Ohio State University in partial fulfillment of the requirements for the degree Doctor of Philosophy.

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ABBREVIATIONS AND ACRONYMS

AAM	- Atmospheric Angular Momentum
ACSM	- American Congress on Surveying and Mapping
AFGL	- Air Force Geophysics Laboratory
AGU	- American Geophysical Union
BIH	- Bureau International de l'Heure
CCRS	- Conventional Celestial Reference System
CDP	- Crustal Dynamics Project
CERGA	- Centre d'Etudes et de Recherches Geodynamiques et Astronomiques
CERI	- Connected Element Radio Interferometry
CORE	- Combined Solution for the Orientation of the Earth
COTES	- IAU/IAG Joint Working Group on the Establishment and Maintenance of a Conventional Terrestrial Reference System
CRS	- Celestial Reference System
CSTG	- Commission on International Coordination of Space Techniques for Geodesy and Geodynamics
CTRS	- Conventional Terrestrial Reference System
DGSS	- Department of Geodetic Science and Surveying
DIS	- (Crustal Dynamics) Data Information System
DSN	- Deep Space Network
EG&G-WASC	- EG&G Washington Analytical Science Center
EGP	- Experimental Geodetic Payload
EROLD	- Earth Rotation from Lunar Distances
ERP	- Earth Rotation Parameters
GPS	- Global Positioning System
GRAS	- George R. Agassiz Station
GSFC	- Goddard Space Flight Center
HRAS	- Harvard Radio Astronomy Station (see GRAS)
IAG	- International Association of Geodesy
IAU	- International Astronomical Union
IERS	- International Earth Rotation Service
IRCC	- Instruction and Research Computer Center
IRIS	- International Radio Interferometric Surveying
IUGG	- International Union of Geodesy and Geophysics
JPL	- Jet Propulsion Laboratory
LOD	- length of day
LLR	- Lunar Laser Ranging
mas	- millisecond of arc
MIT	- Massachusetts Institute of Technology
MERIT	- Monitoring Earth Rotation by Intercomparison of Techniques
MJD	- Modified Julian Date
MLRS	- McDonald Laser Ranging System
NASA	- National Aeronautics and Space Administration
NASDA	- National Space Development Agency of Japan
NEOS	- National Earth Orientation Service

NGS	- National Geodetic Survey
NLRS	- Natmap Laser Ranging System
NNSS	- Navy Navigation Satellite System
OSURF	- Ohio State University Research Foundation
POLARIS	- Polar-motion Analysis by Radio Interferometric Surveying
RGO	- Royal Greenwich Observatory
RRL	- Radio Research Laboratories
RSC	- Radio Source Coordinates
SLR	- Satellite Laser Ranging
TRS	- Terrestrial Reference System
USNO	- United States Naval Observatory
UTX	- University of Texas
VIP	- VLBI Interactive Program
VLBA	- Very Long Baseline Array
VLBI	- Very Long Baseline Interferometry

1. INTRODUCTION

In recent years many new methods of measuring the Earth's rotation have been developed. These include, at the highest levels of accuracy, the space-based methods of Lunar Laser Ranging (LLR), Satellite Laser Ranging (SLR) to Lageos, and Very Long Baseline Interferometry (VLBI). Data from each of these techniques has been obtained and used by many organizations to compute various series of Earth Rotation Parameters (ERP), i. e., polar motion (x_p and y_p) and UT1-UTC values. Generally, these observational systems are viewed as "competing" against each other, each trying to provide the most accurate ERP at the highest time resolution. There have been some attempts to "average" or "combine" the results of the individual systems together and thereby obtain ERP results which contain all of the strengths of the individual systems, but hopefully little of their weaknesses.

However, a closer examination of these procedures causes one to ask a very interesting question: "Do the present methods of combining ERP, by weighted averaging of the parameters determined by the individual systems, cause a loss of ERP information inherent in the observations?" If so, would combining observations from several systems in one adjustment provide better ERP results than averaging the ERP of the various systems, or for that matter, better than the ERP of any single available system? Additionally, one obvious disadvantage of combining data from several systems is that the data must all be in one place for the adjustment to be made. Therefore some comparison should also be made between transmitting data (raw or otherwise), normal equations, or ERP parameters to a central location for use. The latter problem could be called the question of "the effect of data compaction."

In the remainder of this introductory chapter current accuracies for ERP measurement are given, along with a brief summary of the overall scope of this study.

1.1 Current Methods of Earth Rotation Determination

Several methods are currently in use for the determination of ERP, including (in order of development) various optical astrometric methods, Doppler tracking of the NNSS, CERI, VLBI, LLR, SLR to Lageos, tracking of the GPS, and some combination techniques. For an overall review of these methods, see [Moritz and Mueller, 1986]. Some current estimates of the accuracies of these methods are shown in Table 1, with the clear proviso that these are indeed only estimates, with other factors, such as variations, long term drift, availability, and time resolution not being considered here. Many of these estimates are actually based on the comparison of results from various systems. However, it has become generally agreed that any one of the methods such as LLR, SLR, or VLBI is capable of something like several times to up to an order of magnitude improvement over the classical astrometric methods, CERI, and the Doppler tracking of the NNSS. Each observational

Table 1 Current (1985) ERP Accuracies of Various Systems

Observational System	UT1-UTC ms	x_p or y_p mas	Reference
LLR	0.3 0.05 possible	-	[Carter & Robertson, 1985]
SLR or VLBI	0.1-0.6	2	[King, 1985, 1986]
Doppler (NNSS)	-	6-20	[Colquitt et al., 1984; King, 1986]
CERI	4.6 (UT0)	51	[McCarthy et al., 1985]
Optical Astrometry	0.84	11	[Feissel & Zhengxin, 1985]
GPS	2.1	50	[Anderle et al., 1982]
BIH	1.2	7	[Boucher & Feissel, 1984]
CORE	0.42	2.1	[McCarthy & Babcock, 1985]

Values are general estimates, under normal conditions, for approximately one to five days. "possible" indicates insufficient observations yet exist.

system has many advantages and disadvantages relating to factors of accuracy, time resolution, availability of results, operational costs, etc. For example, one discussion of these factors is in [Carter and Robertson, 1985a]. But it has already become quite clear that the methods of SLR to Lageos and VLBI can and are providing the most accurate and highest time resolution ERP. LLR has also demonstrated similar capabilities, but still has yet to reach sustained operation. For a further summary comparing the accuracies of these various techniques, see [King, 1986]. Proposed possible future methods of ERP determination include the use of ring laser gyros [Rotge et al., 1985], or gravity measurements [Wahr, 1985].

Combination solutions (combining the ERP of different systems in some way) have also reached fairly high accuracies. Examples of such solutions are the several combination solutions of the BIH [1986], and the (USNO/NGS) NEOS CORE solution [McCarthy and Babcock, 1985]. These combination solutions routinely incorporate the results of optical astrometry, Doppler tracking of the NNSS, the USNO CERI results, UTX Quick-Look SLR results, LLR results when available, and the IRIS VLBI results. The relative weights of the individual ERP are assigned based on their estimated accuracies and the availability of each of these data types. The primary advantages of these (or any) combination methods are that: first, solutions can be made quickly, based on whatever data happens to be available, and second, it is possible to separate out some of the biases between the various Terrestrial Reference Systems (TRS's) (and possibly the Celestial Reference Systems (CRS's)). This latter advantage makes such solutions important in the possible creation and maintainance of a Conventional Terrestrial Reference System (CTRS) and Conventional Celestial Reference System (CCRS).[1]

1.2 Scope of This Study

At the beginning of this work, the basic problems to be studied have already been presented. They can be summarized briefly in the form of the following two questions:

1. Is there any relative improvement in determining ERP by combining data from various systems in an adjustment, as opposed to combining either the ERP of individual systems or their averaged ERP?

2. What advantages and disadvantages would there be in handling the data (or normal equations, etc.) of these systems, as opposed to handling several ERP series?

It should also be stressed here that in regards to the first question, only the **relative improvement** of the various methods over each other will be considered. We will not be looking so much at the absolute accuracies obtainable, but rather comparing the results of various methods to determine which can provide the best results. In the end it also may not be possible to clearly state which method is "best," but the goal is to provide some measure of the relative advantages of the various methods, so others may decide if further consideration of them or their actual use might be desirable.

After considering these questions, the option to perform a simulation study was chosen, in order to:

1. experiment with different types of data and optimal observation schedules which would not normally be available, but could possibly be made available in reality,

2. use optimal observation schedules which are not currently possible, but which probably will be in the future (especially regarding continuous VLBI observation, or LLR network observations), and

3. specify precisely the input or "real" ERP, allowing us to use it as a standard of comparison for the different methods.

It seems clear that the use of real data or a much more detailed study than this cannot be justified until the possible merit of the method is in fact shown.

Summarizing much of what has been said in this first chapter, the following limitations have been consciously made on this study to keep the work within reason, and yet maximize the results:

---

[1]See [MERIT/COTES, 1985] for definition of these terms.

1. Only simulation experiments will be performed.

2. Only the LLR, SLR to Lageos, and VLBI systems, or combinations of their data or results will be considered.

3. Because of the current importance of very short term ERP determinations, and in order to minimize computer usage and simplify models, only ERP recovery over periods of up to about two weeks will be considered. Investigations into long term drifts (or "biases") in the ERP results will not be done here.

4. Modeled and unmodeled systematic effects will not be considered. This is possible because relative comparisons of the reduction methods are being made and not comparisons of the individual observational systems. If one observing system's observations are degraded by systematic error, then one would expect that all the solutions using those observations would be degraded by the same amount. For example, if the SLR observations were degraded (by for example, refraction model errors), then weighted mean of the ERP, normal equation combination, and grand solutions which contain any of the effects of that data would be degraded by the same amount. However, one obviously cannot compare the observational systems against each other, other than to estimate their best possible accuracies (when no systematic effects are present).

The remainder of this work will be concerned with setting up and looking at the results of the simulation experiments. In Chapter 2, the basic assumptions used to model and adjust the observations of the various systems are summarized. In Chapter 3, the station networks to be used are considered in detail. In addition, the observational targets are defined, the observational schedules presented, and the ERP to be input to the simulations are created. Chapter 4 summarizes the adjustment models as well as the software used in this study. Finally, chapter 5 presents the results of the simulations, both graphically and statistically, with summaries of the results, conclusions, and recommendations for future work presented in Chapter 6.

2. ASSUMPTIONS FOR SIMULATION EXPERIMENTS

The purpose of this chapter is to set out the basic assumptions used in creating the simulation experiments. First, some additional comments (to those in Chapter 1) are made concerning the philosophy used in the simulations, and then each system is discussed in turn, explaining how the "real world" will be modeled. The fine details of station positions, target definitions, observational schedules, instrumental accuracies, etc. are not included here, but are set out in the next chapter, "Input to the Simulation Experiments."

2.1 General Setup of These Simulations

In setting up the simulation experiments of this study, the key feature has been that the overall geometry and first-order gravitational forces under consideration have been carefully simulated. This includes using realistic station positions, instrument types, and instrument precisions, targets for observation (i.e., the Moon in a Keplerian orbit with no librations, Lageos in an orbit with only the central mass and the J_2 (dynamical form factor) effects of the Earth included, and an IRIS radio source catalog), and relatively realistic observing schedules (including an actual IRIS schedule to generate the VLBI observations). The Earth rotation has also been simulated based on actual values from and variations seen in IRIS results.

All other effects are treated as systematic effects which should not substantially affect the results of (comparing) the various simulation solutions. This is true because the same simulated data enters into each of the three types of solutions (individual systems, weighted means of the ERP, and normal equation combination), and adding systematic effects to the data should not greatly affect the relative differences between the results of these solutions.

2.2 Assumptions for the Individual Solutions

In this section, a "simulated world" is created, to model the real one under the guidelines just given. First the common elements of this artificial "universe" are given, and then the elements needed to simulate each observational system are discussed.

The common elements of the simulations for all the systems will be:

1. The station networks are given in a coordinate system (CTRS) with its origin at the same location as the Earth point mass, and"rotating" with respect to a "fixed" coordinate system (CCRS) which is the system of the observational targets. This "rotation" includes the (assumed) perfectly known effects of precession and nutation, and a model for UT1-UTC and polar motion (see the next chapter for the detailed description of this model).

2. Most other common external forces will be ignored, such as those of the Moon, the Sun, and the other planets.

3. Only random error will be considered as existing in the observations (with accuracies described fully in Chapter 3).

4. As discussed at the end of Chapter 1, no systematic effects, such as observational timing biases, refraction, errors in the satellite force model, etc. will be considered.

No attempt will be, or could possibly be, made here to completely describe the observational systems and models required for all three of these modern methods. A good general reference (as already mentioned) is [Moritz and Mueller, 1986]. The detailed description of any one of these systems would be (and each has been several times) a lengthy treatise on its own. Instead some basic knowledge of these systems is assumed, and reference is made to the many sources on these subjects. As examples, for LLR, a general short summary is provided in [Arnold, 1974] and a lengthy description by [Larden, 1982]. Other possible references are [Abbot et al., 1982; Mulholland, 1977; Leick, 1978]. For SLR, program documentation such as [GSFC, 1976; Martin et al., 1976] would appear to provide the best survey of the basic models and methods, and [Degnan, 1985] provides a summary of SLR hardware, etc. For VLBI, an excellent complete survey is provided by [McLintock, 1980] and other valuable sources are [Arnold, 1974; Bock, 1980; Brouwer, 1985; Dermanis, 1977; Herring, 1983; Ma and Zelensky, 1982; Robertson, 1975].

2.2.1 LLR Assumptions

To keep the simulations and especially the software required for them fairly simple, the Moon was considered to be a satellite in a Keplerian orbit about the center of the Earth. This type of model should adequately represent the overall geometry of the Moon's motion over short periods. Two main effects are being ignored under this assumption: first, the Moon's (optical and physical) librations, and second, the use of the various retroreflector arrays on the lunar surface. Shelus [1984, 1985] has indicated that for time periods of up to about two to three weeks, these librations are sufficiently known to have negligible effect on determinations of UT0 (or UT1), and little effect on variation of latitude (polar motion) determinations. Beyond fortnightly, and especially monthly time intervals, the uncertainties in the Moon's physical librations may begin causing systematic error in the LLR results if they are not properly considered. Since the purpose of this study was primarily to look at ERP determinations over short periods (a few hours to several days), this time limitation was accepted, rather then undertake lengthy software development to overcome this specific problem.[2]

Ignoring the distribution of the various lunar retroreflectors appears to likewise be a reasonable assumption. The main purpose in ranging to the

[2]The use of the GEODYN program in LLR mode was considered for the simulation and data reduction, but the GEODYN LLR models were not considered reliable enough to warrant its use [Zelensky, 1984, 1985, 1986].

different retroreflectors is to determine the physical librations and the retroreflector's coordinates [Arnold, 1974, p. 254; Larden, 1982, p. 98]. Six parameters for the Lunar orbit itself will be solved for in all solutions, along with the ERP.

2.2.2 SLR Assumptions

Only the Lageos satellite is considered as a target for SLR (for reasons given in the next chapter). It is considered as a satellite affected only by the central mass and J_2 term of the Earth's gravity field. The J_2 effect has been included since it will realistically cause the regression of the node of Lageos's orbit, nominally maintaining the true station network and satellite geometry for long periods. For the input to the simulations, the orbit itself is defined by measured osculating Keplerian elements of the actual satellite at a given epoch. These elements will be solved for in all cases (along with the ERP, and possibly station coordinates).

2.2.3 VLBI Assumptions

No particularly special assumptions are required for defining the VLBI observations, there being no orbital models to define as with LLR and SLR. As with those systems, the station network is attached to the CTRS. The radio sources are attached to the CCRS, with the right ascension of one source fixed. The remaining positions of the sources will actually be adjusted, but with weights so high that their positions are essentially fixed (in comparison to the adjustment of the Moon's and Lageos's orbits).

Only VLBI delay observations will be considered. All sources consulted [Bock, 1980, p. 42; Bock, 1984; Ma, 1983; Robertson, 1976, pp. 21-22; Zelensky, 1984] indicate that the delay rate observations contribute very little toward determining geodetic parameters such as station coordinates and ERP. They are used in practice since they are simply available as a result of the VLBI correlation process, and since they do allow for the detection and calibration of certain systematic errors [Herring, 1983, section 3.1; Willis, 1985, p. 41]. For the simulations to be done here, they would only serve to double the amount of computer time for the simulation and adjustment of the VLBI data.

3. INPUT TO THE SIMULATION EXPERIMENTS

Now that the models to simulate the real world have been given, these models can be "filled in" by specifying tracking stations and their characteristics, the targets to be tracked, observing schedules, and Earth rotation.

3.1 Station Selection

One of the most important types of input to the simulation experiments is the three observing station networks. The selection of stations having lasers appropriate for ranging to Lageos and/or the lunar retroreflectors, and the radiotelescopes capable of doing VLBI observations is discussed in this section. First, the criteria are established for realistically choosing possible stations, and then the station networks chosen are presented. Finally, comments are made concerning the resulting colocated stations.

3.1.1 Station Selection Criteria

Although several considerations had to be made in choosing observational stations, the basic rule was that the stations chosen must be realistically expected to be in regular operation over the next several years. This implied the following conditions:

1. The station must be in actual operation now or be reasonably expected to be in operation soon.
2. "Observatory type" stations were to be preferred over temporary stations.
3. A station must provide good geometry or coverage in combination with the other stations in its network. Coverage need also not be too dense, as in the case of SLR stations in Europe and the United States.
4. Stations should be chosen which have had or are likely to have colocations to the other station networks.
5. The accuracy of the instruments should be state of the art if possible.
6. Stations which have historically provided very little data should be avoided.

3.1.2 Networks Selected

Following the above criteria, the three networks to be used in the simulation experiments were chosen. The station coordinates used (shown in tables below) are: (a) from the CDP Data Information System (DIS) [Lindner, 1985] for most of the stations; (b) approximate coordinates for stations SIMEIZ, WETZEL, GRAZ, and RGO; and (c) values used by NGS for scheduling VLBI observations for the VLBI stations [Carter, 1984]. The values used are unnecessarily precise for a simulation study, since they were originally obtained to allow eventual real data processing if desired. The possible motion of any of these stations due to plate tectonics will be ignored due to the short time periods being considered in this study, relative to current predictions/measurements of the velocities of these stations.

3.1.2.1 <u>**The LLR Network**</u>. Within the past two years many of the Lunar Laser Ranging (LLR) stations have finally come on line and begun producing good data. Although unfortunately they have mostly been operating after the main MERIT campaign, since June of 1985 their operation does appear to be fairly regular, or able to approach almost daily observations at any time. The final station network selected is described in Table 2, where the UTX Quick-Look Station designation, the NASA station number, the NASA system designation, the location, and the country are given. The locations used for these stations are given in Table 3. Standard deviations of a single (normal point) range are also given in the same table, with the values for MCDON and HOLLAS actually determined from Quick-Look Lageos solutions [Schutz et al., 1985]. The LLR standard deviations are assumed to be similar to those values. All of these standard deviations agree with those estimated by [Dickey et al., 1985]. A world map showing the station locations is also given in Fig. 1.

Table 2 Lunar Laser Ranging Station Network

Abbr. (UTX)	NASA	System (NASA)	Other System Names, Location	Country
MCDON	7086	MCLAS2	MLRS, McDonald Observatory, near Ft. Davis, Texas	USA
HOLLAS	7210	HOLLAS	Haleakala Observatory Maui, Hawaii	USA
ORRLLR	7943	ORRLAS	NLRS, NATMAP, or AUS.FIXED, Orroral Observatory, Orroral	Australia
GRASSE	7835	GRASSE	CERGA or FRA.FIXED, Grasse, Calern	France
RICHMO *	29998 *	RICHMO *	(New) USNO SLR/LLR at Richmond "Polaris" site, near Richmond, Florida	USA
SIMEIZ *	1173 *	SIMEIZ *	(New) "Crimea-1" SLR/LLR, Simeiz, 40 km from Crimean Observatory	USSR

* Designation is unknown or not yet assigned, and therefore the designation shown was used in this study.

Table 3 Lunar Laser Ranging Station Coordinates

Abbr. (UTX)	Latitude/X ° ' " meters	Longitude/Y ° ' " meters	E. Height/Z meters meters	Range S.D. cm
MCDON	30 40 36.743 -1330089.91	255 59 04.330 -5328572.18	1985.4 3236151.02	8.4
HOLLAS	20 42 37.432 -5466018.09	203 44 28.660 -2404114.27	3070. 2242520.57	4.2
ORRLLR	-35 37 35.344 -4447404.24	148 57 12.937 2677175.10	939.0 -3695135.00	5.0
GRASSE	43 45 16.796 4581691.92	6 55 17.806 556199.32	1320. 4389360.35	5.0
RICHMO	25 36 49.520 961259.44	279 36 55.083 -5674091.50	-19.0 2740534.35	10.0
SIMEIZ	44 32 06. 3774888.78	34 01 00. 2547792.60	345. 4450908.30	10.0

Ellipsoid: AE=6378144.11m; 1/f=298.257

Range standard deviations (S.D.) for MCDON and HOLLAS are from [Schutz et al., 1985]. The other standard deviations are estimated.

Of these 6 stations, only the Simeiz and Richmond stations are not yet in LLR operation, although both are in their testing phases. The Simeiz station (named "Crimea-1") was operating in the SLR mode as of August, 1985, and expected to be LLR mode capable early in 1986 [Abalakin et al., 1985]. Some development of an LLR system has also been undertaken by the USNO at GSFC. It was originally understood by the author that this system would probably be moved to the Richmond, Florida, USNO facility, perhaps even in 1986 [McCarthy, 1985, 1986].

3.1.2.2 **<u>The Lageos SLR Network</u>.** The Lageos Satellite Laser Ranging (SLR) station network is given in Table 4, and shown in a world map in Fig. 2. Again, with the exception of the Simeiz and Richmond stations, all of these stations are in regular operation, and Simeiz has just begun operation. In fact, all of the fully operational stations contributed heavily to the total data available from the MERIT main campaign [Schutz et al., 1985; Coates, 1985]. The operating stations are all expected to continue until at least the end of 1991 (when the NASA Crustal Dynamics Project ends), with many of the stations already considered to be in operation permanently. All of these stations are operating or expected to be operating at the state-of-the-art "third generation laser" level, with ranging accuracies of 2 to 15 cm [Schutz, 1985]. Table 5 shows the station coordinates used here along with the currently estimated (normal point) range observation standard deviations of

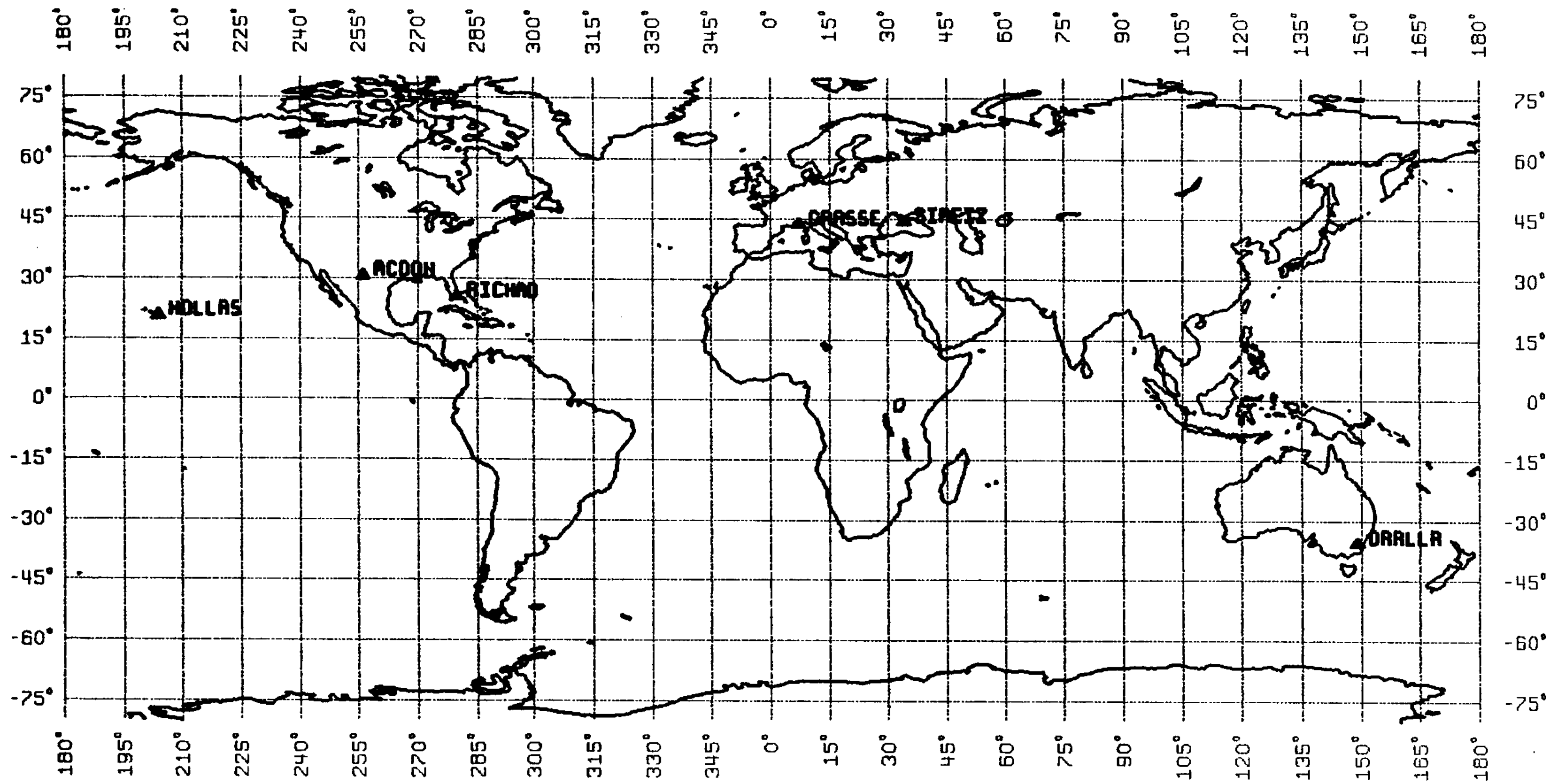

Fig. 1 Lunar laser ranging station network map. Plate Carree projection.
Scale: approx. 1:197 million.

Table 4 Lageos Laser Ranging Station Network

Abbr.	NASA	System	Other system names, Location	Country
YARAG	7090	ML0502	MOBLAS-5, Yaragadee	Australia
GSFC	7105	ML0704	MOBLAS-7, NASA/GSFC,Greenbelt, Maryland	USA
QUINC2	7109	ML0805	MOBLAS-8, Quincy, California	USA
HUAHIN	7121	ML0111	MOBLAS-1, Huahine	Fr. Polynesia
WETZEL	7834	WETTZE	Wettzell	FRG
SHO	7838	SHOLAS	Simosato	Japan
GRAZ	7839	GRAZ	Graz	Austria
RGO	7840	HERSTM	Royal Greenwich Observ., Herstmonceux	UK
ARELAS	7907	ARELAS	Arequipa	Peru
MATERA	7939	MATLAS	Matera	Italy

The LLR stations are all also expected to be ranging to LAGEOS as well. See Table 2.

the instruments, the latter as given by [Schutz et al., 1985] based on the UTX CSR Quick-Look solutions. These accuracies are mostly confirmed (within a few centimeters) by [Coates, 1985].

3.1.2.3 __The VLBI Network__. A basic network of four stations has been chosen as the VLBI network for the purposes of this study. This network includes the "Polaris" station network of the NGS in the U.S. [Bossler, 1982; Carter et al., 1983], and the Wettzel station of the FRG [Schneider et al., 1985]. It is described in Table 6, with station coordinates and a network map being given in Table 7 and Fig. 3 respectively.

There are several reasons for only considering these stations in particular. First, these IRIS [CSTG, 1983, p. 17] stations are the only VLBI stations dedicated to monitoring of the Earth's rotation. All other stations are primarily dedicated to general radio astronomy, spacecraft tracking and communications, and/or baseline determination work. Therefore adequate time is usually never allocated at such stations to make regular Earth orientation observations. The only exceptions are the antennas of NASA's Deep Space Network (DSN), which weekly monitor the Earth's rotation under Project TEMPO [Eubanks et al., 1986; Carter, 1986, pp. 3-4], and the antennas of the Onsala Observatory in Onsala, Sweden, which monthly participates in the IRIS observations (with the four stations previously mentioned). However, the DSN antennas will not be considered here since the observations are usually not done a) on more than one or two baselines, b) as nearly as long and as often as at the IRIS stations, and c) with the highest accuracy (Mark-III) VLBI

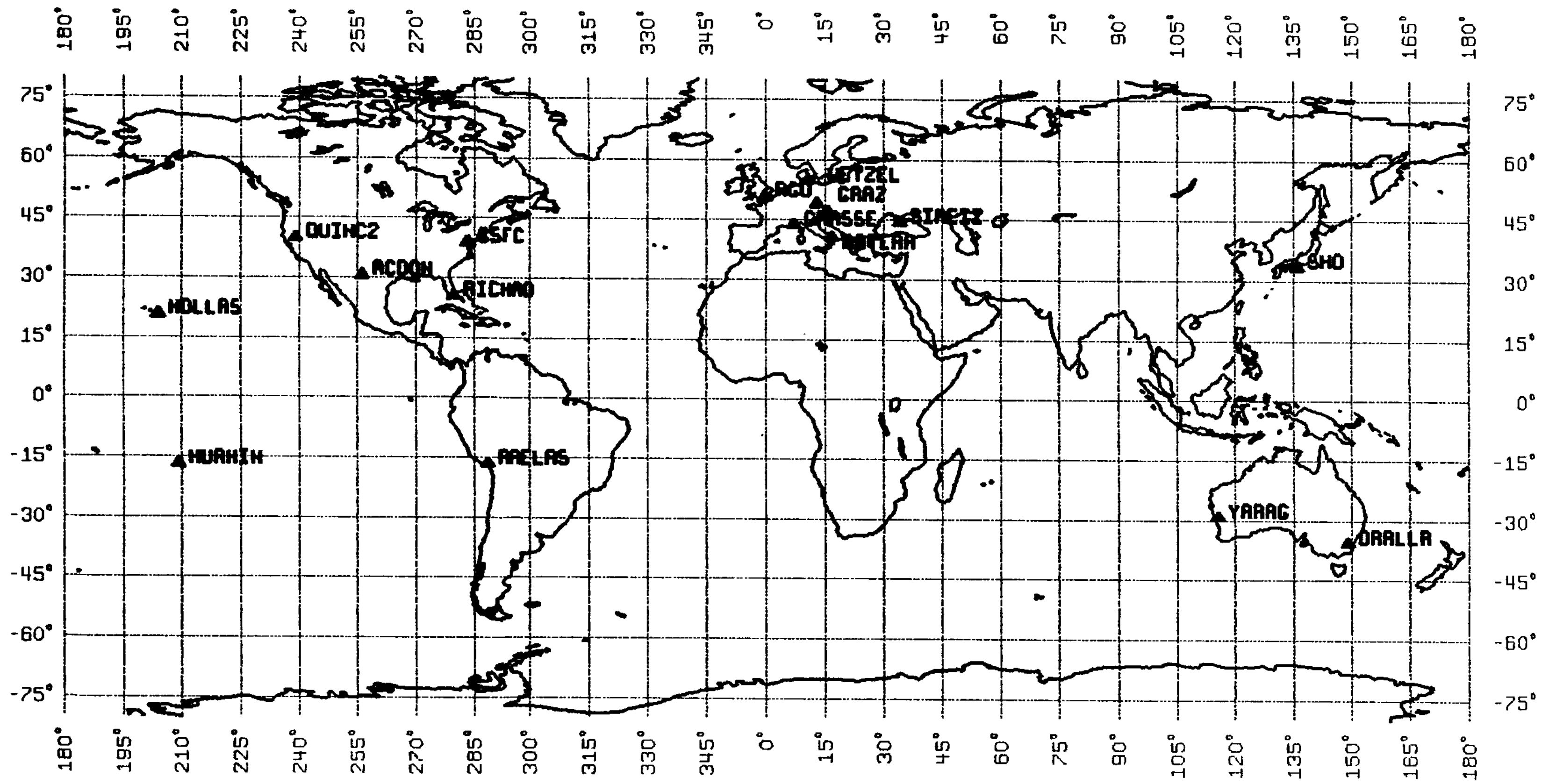

Fig. 2 Lageos laser ranging station network map. Plate Carree projection.
Scale: approx. 1:197 million.

Table 5 Lageos Laser Ranging Station Coordinates

Abbr. (UTX)	Latitude/X ° ' " meters	Longitude/Y ° ' " meters	E. Height/Z meters meters	Range S.D. cm
YARAG	-29 02 51.830	115 20 43.180	284.97	2.3
	-2388872.69	5043368.63	-3078668.90	
GSFC	39 01 13.752	283 10 19.196	54.2	3.4
	1130703.06	-4831396.56	3994122.82	
QUINC2	39 58 30.376	239 03 22.983	1107.65	2.8
	-2517155.01	-4198603.01	4076583.79	
HUAHIN	-16 44 02.610	208 57 40.948	34.20	9.7
	-5345718.92	-2958471.07	-1824680.24	
WETZEL	49 09	12 53	660.	7.1
	4075033.51	932059.79	4801991.00	
SHO	33 34 27.496	135 56 23.537	60.3	9.7
	-3822700.17	3699299.34	3507240.97	
GRAZ	47 04	15 30	540.	3.8
	4194333.63	1163191.66	4647216.74	
RGO	50 52	359 39	75.	4.7
	4033523.72	-24639.70	4924260.20	
ARELAS	-16 27 55.085	288 30 26.814	2486.5	14.5
	1942854.59	-5804072.70	-1796872.68	
MATERA	40 42	16 37	535.	13.9
	4640555.11	1384879.54	4137576.19	

Ellipsoid: AE=6378144.11m; 1/f=298.257

Range standard deviations (S.D.) are from [Schutz et al., 1985]. The LLR stations are all also expected to be ranging to Lageos as well. See Table 3.

receivers (although an upgrade to Mark-III receivers is planned [Edwards et al., 1986]). This latter problem also eliminates observations on a baseline with any IRIS antenna. The monthly observations at Onsala also do not contribute greatly to the IRIS solutions, since they tend to duplicate the geometry provided by the Wettzel station, and do not occur often enough to be significant for purposes of continuous monitoring of ERP. Onsala is also never expected to observe more often than once a month for Earth rotation observations due to the heavy demand on it for general astronomy observations [Carter, 1984]. The main contribution of these additional stations and networks currently is to provide independent checks on the IRIS results, and in serving as a backup to the IRIS observations [Robertson and Carter, 1985] (and in the case of the DSN, providing spacecraft support [Eubanks et al., 1985]).

Table 6 VLBI Station Network

Abbr. (UTX)	NASA	System (NASA)	Other system names, Location	Country
WESTFO	20005	WESTFO	Westford Observatory (NGS "Polaris" station) Westford, Massachusetts	USA
HRAS-0	20007	HRAS-0	GRAS, formerly HRAS, (NGS "Polaris" station) near Ft. Davis, Texas	USA
WETZEL *	29999 *	WETTZE *	Wettzell	FRG
RICHMO *	29998 *	RICHMO *	Richmond Observatory (NGS "Polaris" station) near Richmond, Florida	USA

* Designation is unknown or not yet assigned, and therefore the designation shown was used in this study.

Table 7 VLBI Station Coordinates

Abbr. (UTX)	Latitude/X ° ' " meters	Longitude/Y ° ' " meters	E. Height/Z meters meters
WESTFO	42 36 46.592 +1492208.55	288 30 22.413 -4458131.33	80.93 +4296015.88
HRAS-0	30 38 12.137 -1324207.70	256 3 10.018 -5332028.01	1591.28 +3232118.15
WETZEL	49 8 42.189 +4075533.34	12 52 39.077 +931739.09	659.30 +4801630.53
RICHMO	25 36 49.520 961259.44	279 36 55.083 -5674091.50	-19.00 2740534.35

Ellipsoid: AE=6378144.11m; 1/f=298.257

Coordinates are from an IRIS schedule (see section below on "Observing Schedules").

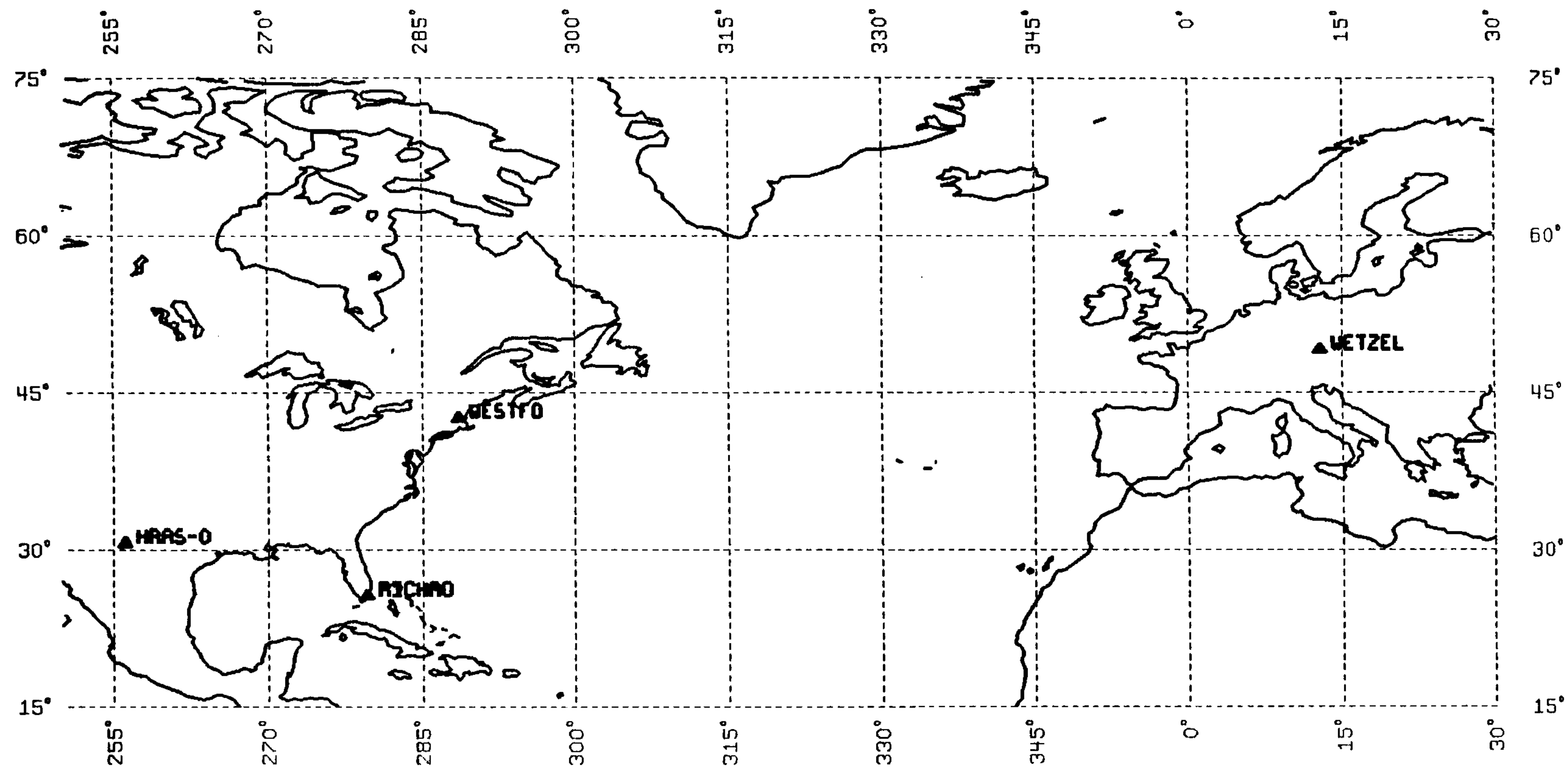

Fig. 3 VLBI station network map. Plate Carree projection.
Scale: approx. 1:77 million.

Secondly, no proposed stations are considered along with the IRIS stations, primarily due to the lack of VLBI schedules to simulate VLBI observations for all participating stations. Such schedules would be very difficult to make up or obtain, for currently nonexistent networks.

A third and significantly interesting reason for only considering the IRIS stations is that they would seem to be nearly necessary and sufficient for the purposes of this study to provide ERP determinations for the VLBI method. Only three of the four stations are actually needed for recovery of the three parameters of Earth Rotation, with the fourth really just providing slightly increased geometric strength and serving as a backup in case of breakdowns, lengthy maintainance, etc. In addition, the present VLBI techniques could not support the addition of many more stations (at least observing regularly in the same network) due to the lack of sufficient correlator time available to process the raw data tapes. Logistical problems also currently exist in terms of having sufficient tape available and in the handling of data tapes, but due to the use of higher density tapes, those problems are expected to be solved soon [Carter, 1986, p. 5].

As to the precision of a single observation (delay) obtained with any pair of the instruments, all estimates available [Carter, 1984; Bock, 1985; Robertson, 1985b; Pascal, 1985, p. 28] indicate that a standard deviation of any delay obtained via Mark-III VLBI receivers would be the same, around 0.1 ns. The actual value used in real VLBI data processing is usually fixed initially at this value, with an adjustment to it made based on the signal-to-noise ratio obtained for the observation at the correlator. Again, for the purposes of a simulation the slight variations due to this will be ignored and a constant standard deviation of 0.1 ns will simply be used.

At the present time, the author is aware of no VLBI solutions which consider the correlations between either adjacent observations (in time) on the same baseline, or observations made on two baselines which have the same station at one of their endpoints. Bock [1983] has shown this to be realistic, and this was also indicated by [Robertson, 1985b]. However, at least one source [Zhi-han, 1985] has indicated that it is possible that the correlations between observations on baselines where the same station is observing may effect ERP solutions. In any case, these correlations would appear to be small enough that they should not noticeably affect the results of any simulations so they will be ignored (set to zero).

3.1.3 Colocations

One of the criteria mentioned above in selecting stations was that stations with other types of instruments would hopefully be nearby, so that colocations could easily be done between such stations. The primary reason for this is that it would allow the (appropriate) constraining of station coordinates in a combination solution and result in a CTRS common to all systems. This would eliminate the current biases (although small) known to exist between the LLR, SLR, and VLBI CTRS's.

The stations chosen assume that colocations could be performed (or already exist) between the stations listed (in groups) in Table 8. However, in all the simulation experiments reported on here, the station coordinates are fixed at their simulation input values, and the constraints implied by the

Table 8 Colocated Stations

System(s)	Abbr. (UTX)	NASA	System (NASA)
LLR/SLR	MCDON	7086	MCLAS2
VLBI	HRAS_0	20007	HRAS_0
LLR/SLR	RICHMO*	29998*	RICHMO*
VLBI	RICHMO*	29998*	RICHMO*
LLR/SLR	GRASSE	7835	GRASSE
SLR	WETZEL	7834	WETTZE
VLBI	WETZEL*	7834*	WETTZE*
LLR/SLR	SIMEIZ*	1173*	SIMEIZ*
LLR/SLR	ORRLLR	7943	ORRLAS
LLR/SLR	HOLLAS	7210	HOLLAS

* Designation is unknown or not yet assigned, and therefore the designation shown was used in this study.

above colocated stations are not used. The colocations are merely mentioned here to further explain the choice of stations. Further simulations could certainly be done in which these colocations could be used, and station positions solved for to see how well (simulated) biases between the individual TRS's (of LLR, SLR, and VLBI) could be recovered.

3.1.4 Fixing Station Coordinates

In all of the simulation experiments (reported on) in this study, the station coordinates have been fixed at their simulation input values. Since we are only interested here in the recovery of ERP, this assumption is reasonable and very similar to actual practice. For example, in most "quick-look" determinations of ERP (at least with laser ranging), station coordinates are usually fixed at their best known values. Occasional "long-arc" solutions are done to solve for the station coordinates (and other geodynamic parameters), but even in this case, the TRS defined by the stations must be in some way held fixed (usually by fixing six coordinates among three stations). The effect of this assumption here is that we ignore errors in the ERP that would be caused by errors in the assumed station coordinates. As just explained in the last section however, any errors due to biases between various TRS's can always be removed using information provided by station colocations (at least at the accuracy level of the colocation measurements). Other errors in the station coordinates can be considered similarly to other possible "systematic errors" that are being ignored in this study, in that in relative comparisons of the various methods, the effects of these errors should mostly cancel out.

3.2 Observation Targets

Now that the discussion of the Earth-based tracking stations (and implicitly, their TRS's) has been completed, the targets to be observed and the TRS's they define must be discussed. Once again, very realistic values (for state vectors and source positions) were used so that a switch to real data processing might eventually be done.

3.2.1 Lunar Laser Ranging—the Moon

For reasons more fully described in section 2.2.1, the Moon has been assumed to act as a point satellite of negligible mass in Keplerian orbit about the Earth. To obtain an appropriate state vector which could be used to define this orbit, a locally written program "PLACOOR" was used to obtain such from the JPL DE/LE 118 ephemeris (provided by B. Putney of GSFC), for the starting time of the simulation experiments. This state vector is given in Table 9.[3]

Table 9 Lunar Orbit Definition

True of date system:	April 5, 1984, 0h UT.
Epoch:	April 5, 1984, 0h UT.
Period:	2412689.2978 seconds
Keplerian Elements:	***State Vector:***
Ω = 10.870397°	X = 230017407.18 m
i = 25.833936°	Y = 296272098.35 m
ω = 154.013215°	Z = 119864520.32 m
a = 388800615.4 m	$\dot{X}$ = -830.37798406 m/s
e = 0.042771136	$\dot{Y}$ = 469.46404827 m/s
M = 254.961219°	$\dot{Z}$ = 299.03004567 m/s

3.2.2 Satellite Laser Ranging— Lageos

The satellite laser ranging technique is of course not limited to observing the satellite Lageos. However, for many well-known reasons, observations to Lageos currently provide the highest accuracy SLR geodynamic results. For example, Earth rotation results from Lageos are currently about an order of magnitude better than those of "its closest competitor," the Starlette satellite [Cheng et al., 1985]. Because of this large difference in sensitivity to Earth rotation between Lageos and all other satellites capable of being laser ranged, and again to keep the simulations from becoming too complex, all other satellites have been ignored.

[3] For a review of the terms and abbreviations used here to define orbits, see [Kaula, 1966, pp. 16-25].

There is promise in the availablity of two new geodynamic satellites, Lageos II [Christodoulidis, 1984] which is now planned for launch in late 1992 [Zerbini, 1986], and Japan's Experimental Geodetic Payload (EGP) [Sasaki, 1984; NASDA, 1983] which was launched in August, 1986. Because of the uncertainty of the launch dates of these or indeed any satellites, and again to keep the simulations simple, these satellites too have been ignored.

In any case, the orbit for Lageos (I) has been defined by starting with one of the monthly state vectors supplied by Mark Torrence of EG&G-WASC and integrating it forward (using the GEODYN program) to provide a final state vector. This integration was done arbitrarily using only the Earth's central mass and J effects, and real ERP (from GEODYN's tables). As long as some reasonably realistic definition of Lageos' orbit is obtained for the desired starting epoch, the method used really should not matter. The orbit definitions used are given in Table 10.

Table 10 Lageos Orbit Definition

Starting (Real) Orbit:	
True of date system:	May 10, 1976, 0h UT.
Epoch:	April 1, 1984, 0h UT.
Period:	13529.708211 seconds
Keplerian Elements:	***State Vector:***
Ω = 94.998827°	X = 1294919.23 m
i = 109.848174°	Y = 10508487.69 m
ω = 4.103997°	Z = 6110317.24 m
a = 12272083.4 m	$\dot{X}$ = 1905.441313 m/s
e = 0.004376275	$\dot{Y}$ = -2875.274098 m/s
M = 27.761723°	$\dot{Z}$ = 4564.495212 m/s
Final Orbit Used to Simulate Data:	
True of date system:	April 5, 1984, 0h UT.
Epoch:	April 5, 1984, 0h UT.
Period:	13525.623240 seconds
Keplerian Elements:	***State Vector:***
Ω = 96.459968°	X = -2114699.0 m
i = 109.809211°	Y = -8622655.4 m
ω = 2.314826°	Z = -8526835.9 m
a = 12269613.1 m	$\dot{X}$ = -1761.9267 m/s
e = 0.004645517	$\dot{Y}$ = 4022.7530 m/s
M = 225.476858°	$\dot{Z}$ = -3603.9154 m/s

3.2.3 VLBI—IRIS Sources

The VLBI CCRS is defined via a given set of radio source coordinates. For these simulations, since IRIS stations and an IRIS observing schedule are being used (see below), the radio source catalog is by default that used with that schedule, i.e., the 14 sources in use by IRIS (at the time that schedule was made up).[4] Their designations in IAU format, their common names (if any), and their right ascensions and declinations are given in Table 11. These coordinates are so well known from observations specifically tailored to determine them that they generally can be held fixed or at least assigned very strong weights in adjustments to obtain geodynamic parameters [Zelensky, 1985]. In any case, the right ascension of one of the quasars, usually 3C273B, is fixed to establish the origin of the right ascension system. This defines the VLBI CRS orientation, and in turn in any combined solution with LLR and/or SLR would establish the orientation of the overall CCRS, provided that the lunar and satellite orbits are not fixed. In the simulation experiments done for this study, this procedure is followed, with the 3C273B right ascension fixed, and the other source coordinates given very strong weights, with standard deviations of 5 microseconds in right ascension and 50 microarcseconds in declination.

This IRIS radio source catalog is already in many ways an optimum catalog, and it would be difficult to choose a better one even if we were not limited by the schedule chosen. Given the choice of stations made, these sources provide a fairly uniform coverage over the observable sky. Such a distribution should provide a good geometry for the recovering of geodynamic parameters and of various biases (such as clock effects) [Willis, 1985, pp. 46-47; Bock, 1980]. Other source catalogs, such as that of the CDP or the DSN, tend to be tuned to the specific stations and instruments for which they are used. Additionally, the use of too many sources causes a loss of sensitivity to some biases (as each specific source is observed less and less) and less overall observing time, due to the time spent by the radiotelescopes slewing between sources [Bock, 1980, pp. 65-69].

Table 11 VLBI (IRIS) Radio Source Catalog

Mean System: B1950.0

IAU No.	Common Name	Right Ascension h m s	Declination ° ' "
0106+013		01 06 04.51808	+01 19 01.0740
0212+735		02 12 49.87743	+73 35 39.6825
0229+131		02 29 02.52	+13 09 40.4
0528+134		05 28 06.75	+13 29 42.6
0552+398		05 52 01.37323	+39 48 21.9237
0851+202	OJ287	08 51 57.229751	+20 17 58.595671
0923+392	4C39.25	09 23 55.294295	+39 15 23.828283
1226+023	3C273B	12 26 33.2460000	+02 19 43.470497
1404+286	OQ208	14 04 45.6284	+28 41 29.5239
1641+399	3C345	16 41 17.640138	+39 54 10.991065
1803+784		18 03 39.34961	+78 27 54.3584
2134+004	2134+00	21 34 05.226113	+00 28 25.020373
2200+420	VR422201	22 00 39.387971	+42 02 08.330657
2251+158	3C454.3	22 51 29.533525	+15 52 54.183708

[4] A check of *IRIS Bulletin B*, No. 30, Oct., 1986, shows that these sources were still the ones in use as of Aug., 1986.

Finally, it is recognized that with time, some radio sources change their strength or even their structure. Slight changes are accordingly made in any source catalog, usually by dropping the problem source and adding a new one nearby [Carter, 1984; Vandenberg, 1986]. Such changes should have little effect on ERP determination, and in fact hopefully negligible effect on any geodynamic results.

3.3 Other Observation Criteria

So far, the assumed stations and instruments as well as the targets to be observed have been described. In this section, the way in which the instruments observe, i.e., when and for how long, will be considered.

3.3.1 Time Period of Observations

As to the **epoch** of when the simulation experiments will take place, the choice is quite arbitrary. As long as the targets and the stations are all given with their true geometry for the chosen time, the epoch itself is unimportant. The beginning of the MERIT Intensive Campaign, April 1, 1984, was actually chosen for two reasons:

(1) Since this study was first under consideration in 1984, it was then thought that the intensive campaign would provide very high data rates, which would allow this study to be undertaken with real data *if* eventually desired. The results could then be compared with the simulations over that same period. In actual fact, the SLR and VLBI data rates were indeed fairly high, but there was negligible LLR data obtained. Of course at present, if real data were going to be used, a later period with more intensive SLR observations and at least some significant LLR observations would be chosen.

(2) The variations in the ERP during that period were fairly rapid and well measured by SLR and VLBI, and could be used to create in some way the "input" ERP to the simulation experiments.

The Moon was new on March 28, so April 5, 1984 was used as the starting date, in order to avoid a lack of LLR data (which occurs in reality near new Moon and cannot be avoided). Obviously, this lack of LLR data once a month for a few days has always been a problem with LLR, but due to the short periods under consideration in this study (see just below), this limitation will not be considered to be a major problem.

As to the **length** of the time periods used in the simulation experiments, only short periods, specifically up to 15 days will be considered. This is primarily because of both the current need for an increase in accuracy of ERP for determinations in or over short time periods, and due to problems in the simulation experiments which would become significant over longer periods.

The current needs in ERP determination are for the highest accuracies and time resolutions possible. This means not only that the ERP should be measured in very short time periods, so as to take a "snapshot" of them, rather than obtaining an average value, but also that the values are needed quickly. For example, the higher density ERP can be used in geophysical studies such as of the AAM or of tides, nutation, etc., and the fast determination of ERP is important for use in defining the orientation of satellite reference systems, such as for the GPS coordinate system [McCarthy, 1986]. Current methods already appear capable of measuring with very high accuracy anywhere from one- to ten-day average values, at least after substantial data processing is completed. This supplies values which are already adequate for geophysical studies of long period phenomena. Certainly, the combination of data from different systems may result in improvement here, but is is not as important as at shorter periods of time.

Although it would be nice to simply extend the end times of the simulation experiments so one could look at results over long periods, to do so would be very difficult and to some extent meaningless for two reasons:

(1) The amount of computer time to simulate and adjust (several times) large amounts of data (with perhaps thousands of observations per day) certainly becomes significant with time periods over a few weeks.

(2) As the total period of time covered increases, some of the effects which have been ignored (such as those on the Moon's and Lageos' orbits) become significant enough that if considered they would probably begin affecting the relative results of the simulation experiments.

Therefore, due to the importance of improving at least the short-period ERP determination, and due to the above problems with longer periods, only periods up to 15 days were considered. This would have allowed comparing of at least three sets of five-day values if necessary, with five-day values being for many years the common values determined.

3.3.2 Observational Data Rates

The rates of observation by the various systems are assumed the same as their real (at least normal point) values. For LLR, this means one normal point every ten minutes [Abbot et al., 1973; Shelus, 1985]. For SLR, one normal point every two minutes is assumed (two- or three-minute normal points are commonly used). For VLBI the rate is determined by the observing schedule, but, e.g., IRIS observations last two or four minutes per source, depending on their signal strength, with antenna slew time allowed for between observations.

Normal points are used rather than full-rate data for LLR and SLR, so that the amount of computation time is minimized, and so that the high correlations between full-rate data values can be ignored. For a simulation, their use should be quite justified, since their use is adequate for most purposes even with real data. In reality, full-rate data is also much more difficult to transmit and process for use in fast ERP determinations.

3.3.3 Observing Schedules

For most of the simulation experiments, it was planned to use a simple rule for determining when observations should be made: observe whenever possible. In other words, the LLR and SLR instruments would operate whenever their targets, the Moon or Lageos, were above the horizon. The VLBI network would operate continuously, repeating the same 24 (sidereal) hour schedule over and over.

Obviously, at first glance this is not a very realistic schedule. However, for the purpose of comparing the results obtained by these individual systems with various combined solutions results, and for determining the best possible accuracies obtainable by these systems, such experiments should be quite reasonable.

As to the continuous operation of these networks ever being possible, this is unlikely, except over specific "intensive campaign" periods when such schedules may be possible, although data loss from the laser systems would be inevitable due to weather conditions. However, assuming that adequate manpower was available (and that the problems of LLR use throughout most of a lunation were solved) continuous LLR/SLR operation is realistic. To the author's knowledge, all of the laser systems are currently daylight operation capable (except for the LLR problems). As to VLBI, the main problem is having an adequate supply of tapes and correlator time available. As these problems are rapidly being solved, continuous VLBI at least occasionally for several days is quite possible [Carter, 1984].

The VLBI observations are defined by an actual IRIS schedule, supplied by Dr. Carter [1984] and J. Ross MacKay of NGS. The 24-hour schedule ("B4") was originally made up for IRIS observations which were to take place during the period September 30 - October 2, 1984. This schedule was in the VLBI Mark-III system "SKED" program format, as defined in [Vandenberg and Shaffer, 1983]. To use it on a given date, a locally written program "SKEDVIP" reads the schedule and advances it as many sidereal days as necessary to obtain a schedule for the specified date. The creation of the simulated VLBI data from this new schedule is described in the next chapter. Finally, to realistically include only observations which would not be badly affected by unmodeled tropospheric refraction, the commonly used cutoff angle of 15 degrees was enforced when simulating the LLR and SLR data. The VLBI data cutoff angle is automatically implied by the schedule being used.

3.4 ERP Simulation

The last type of input which is needed for the simulation experiments are values for the polar motion and UT1-UTC. On the one hand, once again, these values should be as realistic as possible. On the other, their creation should be relatively simple, and they should probably not contain the sharp peaks and valleys or "noise" encountered in real high resolution ERP series.

Several possible representations of the ERP have been considered. One possibility is the use of simple step functions, as in other simulations to recover ERP [Dermanis, 1977, pp. 6-10; Pavlis, 1982, pp. 150-163]. Appropriate harmonic functions have also been used [Baraka, 1983, pp. 10-15]. Another possiblity is to create appropriate curves by superimposing periods and amplitudes (possibly obtained from spectral analysis) known to exist in ERP data. Using real data, or smoothed values is another possibility, but as just indicated real data either tends to have too many strong fluctuations due to random and systematic errors, or if smoothed, may not have the variations present which appear to exist in reality.

For this particular study, high resolution simulated ERP values were clearly needed as the intention was to look at the recovery of such values on time periods of around a day or less. In fact since it was desired to look at the recovery of ERP with different time periods for the parameters, simulated ERP with intervals of six, 12, 24 hours, two, three, or five days have been created. Another complication was that the particular software in use only allowed input of the simulated values and recovery of them as step functions (possibly of different time resolution).

3.4.1 Model Used for ERP Simulation

Considering the above alternatives and requirements, a simple procedure was developed to simulate ERP values using combinations of some of the methods mentioned. The method developed includes simple interpolation of a given real ERP series (which has values at multi-day intervals), and then the superimposition of a sine curve of appropriate amplitude and period onto it. From the resulting curve, high resolution step functions are created to provide the input ERP, and step functions of low resolution can also be created to compare with ERP values recovered at that resolution.

This procedure is broken down and explained in the following steps:

(1) A time interval, Δt, is chosen as the length of each simulated ERP step. Start and end epochs (t_b, t_e) are also chosen. The first step is defined with the period $t_b - \Delta t$ to $t_b + \Delta t$ with the central time t_b being the moment at which each parameter is considered to have been determined.

(2) A set of real ERP values is then interpolated at the time t where $t \in (t_b, t_b + \Delta t, t_b + 2\Delta t, \text{etc.})$. This interpolation could have been done in any reasonable way (e.g., with polynomials or splines), but again for simplicity was done linearly. The software developed for this is capable of reading any of the ERP series in the BIH format, as provided on magnetic tape (in June, 1985) by M. Feissel of the BIH. An IRIS ERP set, ERP(NGS) 83 R 01, was actually used.

(3) Next, reasonable period and amplitude sine curves are determined from real data. In this case, the ERP plots in [Robertson et al., 1985; Robertson and Carter, 1985, pp. 302-305; Carter and Robertson, 1985b] were examined to obtain possible values. These particular curves were used since a) they represent the highest time resolution ERP values yet published, and b) they demonstrate that comparable values were obtained via various observing networks and methods. The latter point shows that the ERP variations are likely real and not due to random or some type of systematic error. These curves are of the form:

$$V_i^j = C_i^j \sin\left(\mathrm{mod}\left((t-t_b) / P_i^j,\ 2\pi\right)\right) \qquad (1)$$

where

V_i^j – value to be added to the interpolated ERP (V_i^o)

j – number of curve (j=1 only here)

i – ERP type, 1 => x_p, 2 => y_p, and 3 => UT1

C_i^j – amplitude of sine curve j, for ERP type i

p_i^j – period of sine curve j, for ERP type i t   - time of interpolation in MJD

t_b - starting time in MJD

and the selected coefficients for the curves are given in Table 12. Note that three such curves in all were used, with one for each of the three types of parameters. The software developed could handle really any number of such superimposed curves, but one set seemed to provide enough variation in the simulated ERP.[5]

Table 12 Coefficients of Superimposed Curves Used to Simulate ERP

ERP Type	Curve No.	Amplitude (mas or ms)	Period (days)
(i)	j	C	P
x_p	1	10	2.8
y_p	1	10	1.8
UT1-UTC	1	2	1.1

(4) Finally, for each parameter and time, the interpolated value (V_i^o) and the values obtained from the sine curves (V_i^j) are added together to obtain the final simulated value (V_i). Using the time period under consideration (April 5 - April 20, 1984; see above under Section 3.3.1) to obtain the interpolated values from the IRIS ERP and the Table 12 coefficients for the superimposed sine curves, the time series for x_p, y_p, and UT1-UTC are obtained as plotted in Figs. 4 and 5. Fig. 4 shows a plot of all the simulated values over the 15 days, while Fig. 5 shows only the shorter period ERP over the first four days so that their variation can be more clearly seen.

As shown by the plots in Figs. 4 and 5 of different time resolution ERP, this method has the advantage that further ERP series can be created simply by averaging the values already obtained over the new periods. These new periods would of course have step sizes that are even multiples of the original Δt (i.e., now Δt = 12, 24 hours, 2, 3, and 5 days), and these values could then be compared with the recovered ERP values over these same steps.

At the same time that these series are being created, average values of the ERP over the entire period t_b to t_e are created. These values along with very loose weights (large standard deviations) are used as approximate values for the recovery of the ERP values. If solutions with real data were being done, much more accurate ERP values (either predicted or from simple preliminary solutions) would usually be available. Along with these approximate values, standard deviations of 10 mas for polar motion and 3 ms for UT1-UTC were used to weight the ERP in all adjustments.

[5] It was originally planned that higher frequency curves would additionally be used. However, it was not realized until after all experiments were completed that with the periods used (2 and 12 hours) the values to be added went to zero at the six-hour intervals chosen.

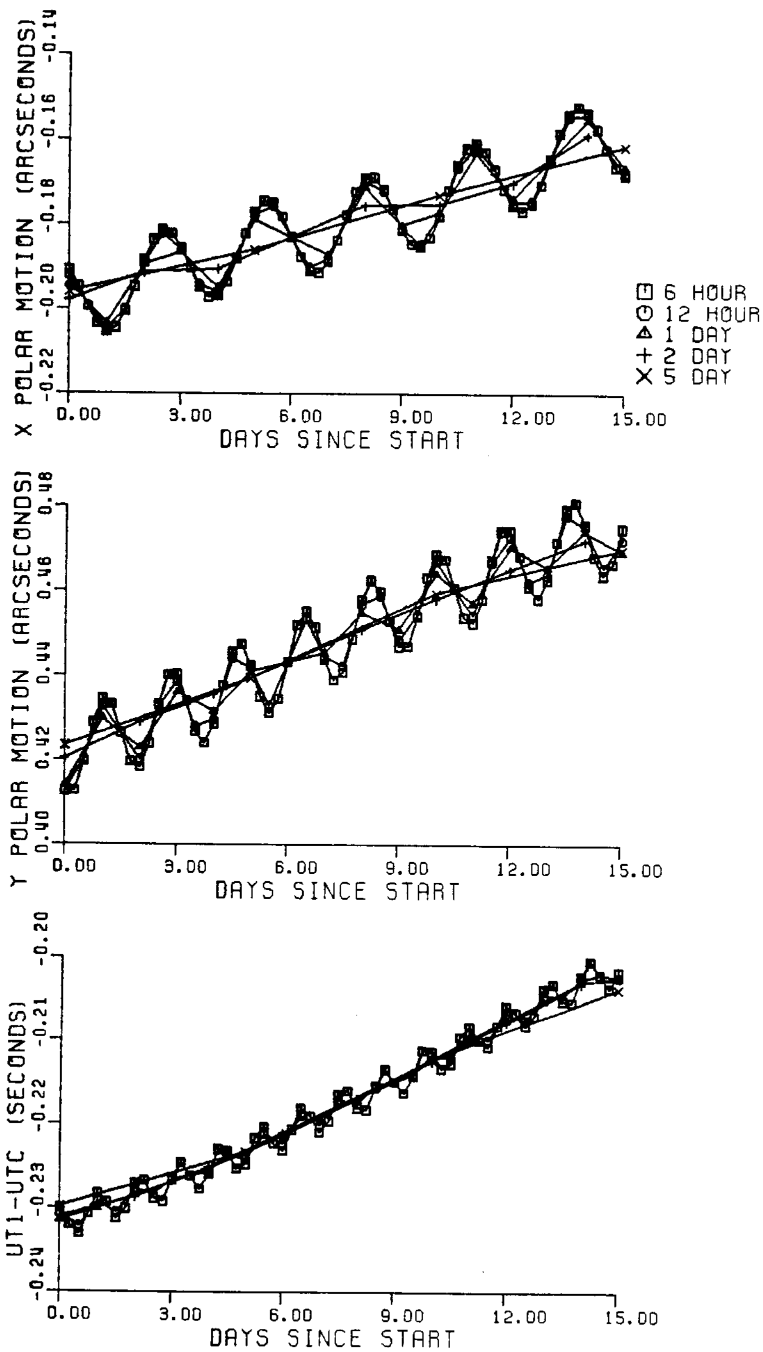

Fig. 4 Simulated ERP curves over 15 days. Symbols for each time series are connected by lines for clarity.

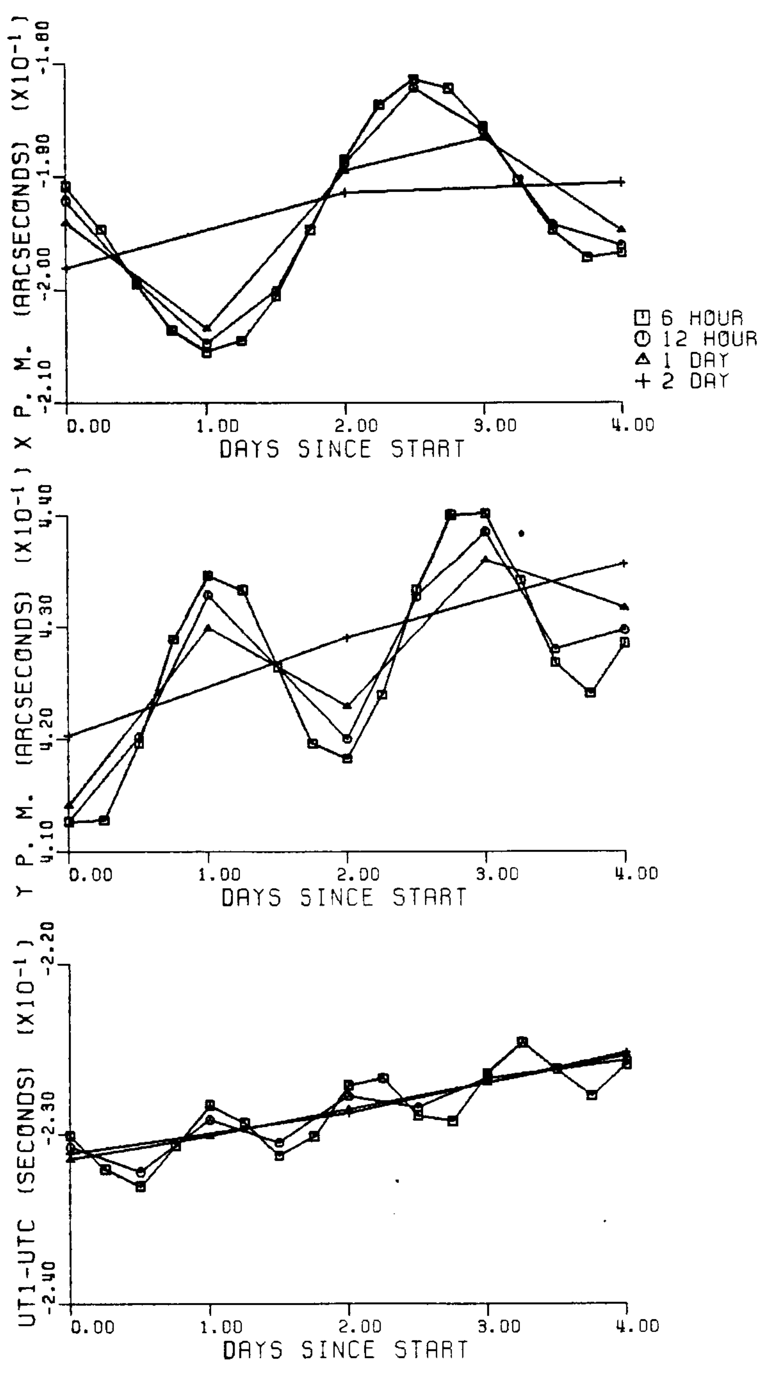

Fig. 5 Simulated short-period ERP curves over first four days. Symbols for each time series are connected by lines for clarity.

4. MATHEMATICAL MODELS (AND SOFTWARE) USED IN THE SIMULATIONS

Before proceeding on to present the results of the simulation experiments in Chapter 5, it is appropriate to present a review of the adjustment models used in creating and adjusting the simulated observations. A review is also given here of the software used in this study, both for the simulation and adjustment tasks and for other miscellaneous problems.

4.1 Simulation and Adjustment of the Observations

In this section a general description of the overall adjustment procedure is given. This includes first a description of the observation equations for laser ranging and VLBI. This is followed by a description of the method used for orbit prediction, necessary in the reduction of the laser ranging observations. Next the adjustment model itself is presented in order to review how the solved for parameters (in this case ERP, state vectors, and station coordinates) are obtained by adjustment of the observations. Finally, various ERP combination methods are described: combination of the ERP of different systems by weighted means, combination of normal equations of different systems to obtain ERP, and adjustment of all data together to obtain ERP.

Generally, the actual simulation of the data will not be discussed throughout this section, but only with the observation equation discussion, since those are the basic equations used for the simulation of the data.

4.1.1 Observation Equations

Complete discussion of the formation of the observations for SLR and VLBI are given in many references. For satellite ranging, see for example [Kaula, 1966, pp. 78-81; GSFC, 1976, pp. 7-11 to 7-15]. Due to the assumptions previously made for the simulations, LLR can be treated as a special case of SLR. In reality, much more detailed observations equations are used in practice. See, e.g., [Arnold, 1974, pp. 249-255; Larden, 1982, pp. 69-90]. For VLBI, derivation of the observation equations is given in [GSFC, 1976, pp. 7-41 to 7-42; McLintock, 1980, pp. 25-27; Bock, 1980, pp. 9-17].

In this particular review, the prime references considered are [Martin et al., 1976, section 6.0 and 6.1] for the ranging, and [Ma and Zelensky, 1982, section II] for the VLBI, as these are the prime sources with the descriptions of the models in the software (GEODYN) actually used for the data simulation and some of the adjustments.

As regards the system of astronomical constants and equations in use in these derivations and in the simulations, it should be noted that only the pre-1980 models are in use, primarily because of the software selected. In addition, some of the values described as "computed" below, such as nutation and the equation of the equinox, are obtained from a JPL planetary and lunar ephemeris by the software actually in use. None of this should affect the results of the experiments, since only the approximate geometry of the stations and targets is to be maintained. What is critical of course (and carefully done here) is that the same geometric models be used for the simulation and adjustment of the data.

4.1.1.1 **LLR and SLR Observation Equations**. As just mentioned, the LLR and SLR observations will be considered at the same time, since they are treated nearly identically in the simulation experiments here (the only difference in the observation equations being that the initial state vector of the "satellite" will of course be different).

We will express an observation equation in the form:

$$l_i = f(x_i) + v_i \tag{2}$$

where l_i is the value of any particular observation, f is a function of parameters x_i such as for Earth rotation, and satellite and station positions (and in reality many other biases). v_i is the discrepancy of the measured and modeled observation and is the result of random error in the measurement l_i and the incompleteness of the model f.

For satellite ranging, this equation is of the form:

$$l_i = |\bar{\rho}| + v_i \tag{3}$$

where $\bar{\rho}$ is the slant range vector from the observing station to the satellite. Although the actual measurement in laser ranging is the round trip light travel time, this is usually expressed as one-way range. $\bar{\rho}$ may also be expressed as:

$$\bar{\rho} = \bar{r}^T - \bar{r}_{ob} \tag{4}$$

where $\bar{r}^T$ is the position vector of the satellite (X^T, Y^T, Z^T) in the TRS being used, and $\bar{r}_{ob}$ (X_{ob}, Y_{ob}, Z_{ob}) is the station position vector in the same TRS.

However, since the prediction of the satellite's state vector (X, Y, Z, $\dot{X}$, $\dot{Y}$, $\dot{Z}$) is done in an inertial system (see section 4.1.2) we need to give $\bar{r}^T$ as

$$\bar{r}^T = S\ N\ P\ \bar{r} \tag{5}$$

where N and P are transformation matrices to account for nutation and precession (and are assumed known here), and $\bar{r}$ is the inertial position vector of the satellite. S includes the total effect of Earth rotation and is a function of sidereal time and the ERP:

$$S = R_2(-x_p)\ R_1(-y_p)\ R_3\ (GAST) \tag{6}$$

See [Mueller, 1969, pp. 59-86]. In this equation, R_i are rotation matrices [Mueller, 1969, p. 43]. GAST is the "Greenwich Apparent Sidereal Time" and is expressed by:

$$GAST = GMST \text{ at } 0^h \text{ UT} + \dot{\theta}\,(t_{DF} + UT1\text{-}UTC) + Eq.\ E \tag{7}$$

and of course, x_p and y_p, and UT1-UTC are the ERP themselves. GMST at 0^h UT is obtained from Newcomb's Equation [ESAENA, 1977, p. 75], $\dot{\theta}$ is the mean rate of the advance of GMST per day (which can be obtained from the time derivative of Newcomb's Equation), t_{DF} is the day fraction (in UTC) of the time of observation t, and Eq. E. is the equation of the equinox, which is given by the expression:

$$Eq.\ E = \Delta\psi \cos \varepsilon \tag{8}$$

[ESAENA, 1977, p. 43] where $\Delta\psi$ is the nutation in longitude (already computed in conjunction with the formation of the N matrix), and ε is the obliquity of the ecliptic, as given in [ESAENA, 1977, p. 38].

Back substituting these equations gives:

$$l_i = |\, R_2(-x_p)\, R_1(-y_p)\, R_3(GAST)\, N\, P\, \bar{r} - \bar{r}_{ob} \,| + v_i \tag{9}$$

(with GAST given by equation 7) as the final observation equation for ranging observations.

4.1.1.2 <u>VLBI Observation Equations</u>. The VLBI observable is geometric time delay, Δt, the difference in time between the arrival of a given radio signal at station 1 from that at station 2. Once again, since the effects of systematic error mostly cancel out when comparing the results of the simulation experiments, errors in the station clocks or delays introduced by the Earth's atmosphere are not considered. The observation equation then becomes simply:

$$\Delta t_i = -\frac{B(t) \cdot \hat{S}}{c} + v_i \tag{10}$$

where

- i is still the observation number
- $\underset{\sim}{B}(t)$ is the baseline vector from station 1 to station 2
- $\hat{S}$ is the unit vector of the radio source
- c is the speed of light

Rigorously, the baseline vector B(t) is actually determined by successive approximations, since it is measured over the time t to t + Δt. See [Ma and Zelensky, 1982, section 1B]. Since the baseline vector is in the earth-fixed system (TRS), the radio source position is also needed in this system and is obtained from:

$$\hat{S} = S\ N\ P\ \hat{S}_{1950.0} \tag{11}$$

analogous to equation 5, where $\hat{S}_{1950.0}$ is the unit vector of the radio source position at a reference epoch and equinox, usually (and here) B1950.0. As just described following equation 5, the ERP are included in this last equation in the S matrix (and therefore may be solved for in any adjustment using these equations).

4.1.1.3 <u>**Simulation of Observations**</u>. Simulation of observations is done by taking: a) the approximate values for station positions, b) the approximate values for the ERP at the observation time, and c) the predicted positions of the satellite (Lageos or the Moon) at the observation time, and substituting these values into the original nonlinear observation equations. The resulting "perfect" observations correspond to the "computed" values of the adjustment process.

These "perfect" observations alone may be useful. Checking of software, or checking for numerical or round off errors can be done by seeing how well the approximate values can be recovered, using these "perfect" observations in an adjustment. Additionally, since all natural phenomena are continuous, and yet observations of them are always at discrete times, adjustment of such "perfect" observations also may indicate when this discreteness of the observations causes problems in recovering the desired parameters.

In any case, for all of the simulation experiments described here, random noise was added to the observations, with a mean of zero (no "systematic error") and a standard deviation appropriate for the observing system (see 3.1.2 above). This noise is generated for the range observations (LLR and SLR) in the GEODYN software itself when the observations are simulated. For the VLBI observations, it is generated by the VLBISIM program which creates the simulated VLBI data. See section 4.2 for further information on the software.

4.1.2 Orbit Prediction

It is appropriate to review here generally how the satellite position at any moment is obtained and how partials of the observation equations with respect to a satellite state vector are computed. This discussion will follow that of [Martin et al., 1976, section 2.1].

First, the parameter set to be solved for, $\bar{X}$ is divided into two sets, a set $\bar{\alpha}$ and a set $\bar{\beta}$, depending on whether they are independent of the satellite's dynamics. Partial derivatives of the observation equations (9 and 10) with respect to the $\bar{\alpha}$ parameters may be directly computed at the given observation times t. These $\bar{\alpha}$ parameters include the ERP, station coordinates, and (with real data) measurement biases.

The partial derivatives with respect to the solution parameters $\bar{\beta}$ (which is usually a state vector for the satellite at time t_0), are computed via the chain rule:

$$\frac{\partial l_i}{\partial \bar{\beta}} = \frac{\partial l_i}{\partial (X,\ Y,\ Z,\ \dot{X},\ \dot{Y},\ \dot{Z})_t} \quad \frac{\partial (X,\ Y,\ Z,\ \dot{X},\ \dot{Y},\ \dot{Z})_t}{\partial \bar{\beta}} \tag{12}$$

where l_i is the observation equation being differentiated. The complete set of expressions:

$$\frac{\partial (X,\ Y,\ Z,\ X,\ Y,\ Z)_t}{\partial \bar{\beta}} \tag{13}$$

are called the variational partials and are obtained by integration of the variational equations (see below). This integration is done along with the integration of the equations of motion.

The equations of motion express the position and velocity of the satellite at a given time, under the accelerations introduced by outside forces. As previously mentioned, for LLR only the Earth's central force (due to GM) is considered, while for Lageos, the dynamical form factor of the Earth (J_2) is also taken into account. The variational equations are simply the derivative of the equations of motion with respect to β. See [Martin et al., 1976, section 8.2].

Since only effects up to J_2 were being considered, the integration of the equations of motion and variational equations could have been done analytically. However, the possibility of using real data led to the use of software which does the solution via numerical integration. An 11th-order Cowell predictor-corrector method is used [Martin et al., 1976, section 9], with a step size of two minutes for Lageos and ten minutes for the Moon.

4.1.3 Adjustment Process

The statistical estimation procedure for the solution of the parameters is a partitioned Bayesian least squares method. In the terminology of [Uotila, 1986], this would be known as a weighted least squares observation equations model with two or more (uncorrelated) sets of weighted parameters. Since the relation between the parameters and the observations (equations 9 and 10) is nonlinear, iterative techniques may be necessary to solve the resulting nonlinear normal equations. The details of this adjustment model and its derivation are given in [Martin et al., 1976, section 10; GTDS, 1976, section 8]. Similar models are also presented in [Uotila, 1986, pp. 95-96, 103-104]. The nonlinear observation equations used to simulate the data have now been linearized in order to form the normal equations. Approximate values are also the same as the parameter values used to simulate the data, except for the approximate ERP values, which are now given as fixed average values of the simulation input values (see the end of the last chapter). The parameters are weighted as previously described, for the orbit parameters, radio source coordinates, station coordinates, and ERP.

The separation of the parameters into two sets is done on the basis of whether there is a connection with the satellite orbit. The satellite state vector at epoch t_0 (and for real data, other satellite force model parameters) are part of one set of parameters, called the "arc set." One such set exists for each initial state vector being estimated. Parameters such as ERP (a step function time series of x_p, y_p, and UT1-UTC), station coordinates (and station biases when using real data) are part of another "common" set of parameters. In practice, it is likely that some unmodeled systematic errors (e.g., due to refraction model errors) may exist in real observational data, and that the observations of the various observing systems and therefore the parameters of the various arc sets are correlated. However, these correlations are expected to be small, and they are neglected here since a) their values cannot be easily determined, and b) this assumption enormously simplifies the computations. Maximum possible values could always be estimated for these correlations and further simulations done to see their effect if desired.

The iteration scheme in use to solve this type of normal equations is one of "inner" and "outer" iterations. If the approximate values of the common set parameters are reasonably close to their values and appropriately weighted, the adjustment may be iterated upon each "arc set" of parameters as many times as necessary first (these being called the "inner" iterations). When the solution for all the "arc sets" of parameters has each converged, the common parameters are finally solved for. This procedure may also be iterated, and is called an "outer iteration." This procedure will converge to the same results as if a single general iteration solution is done. However, less computation is actually required, and with real data, bad data sets for a given arc can easily be identified and taken care of before contaminating the common parameters. This partitioning of the iteration is only possible and is most useful because of the just-mentioned assumed noncorrelation of the arc parameter sets with each other.

4.1.4 Combination Solutions

Various methods exist and are already in use, in which ERP values obtained from different systems are combined in some way to obtain a new set of "combined" or "averaged" ERP values. But the other obvious procedure, of directly combining observations from different systems in an adjustment, is the method primarily being investigated in this study. Indeed, most of the results presented in the next chapter will look at the comparison of the ERP values input to the simulation, the ERP recovered individually by each system, the ERP recovered by combining the normal equations of the individual system solutions, and the ERP recovered in one grand solution with all the data.

In this section, this normal equation combination method will be briefly explained, but first the two other combination methods will be considered. These two methods are first, the combination of the individual systems' ERP values by weighted means, and secondly, the grand solution of all data in one adjustment.

4.1.4.1 **Combination of ERP by Weighted Means.** This method is described fairly well by its title. First, an ERP series for each system is obtained by the independent adjustment of that system's data. The weighted mean of these series is then taken at each ERP epoch and a new series is obtained. The weights of course are taken as the inverse of the variances obtained for the ERP values, with the correlations between parameters being ignored. This is because in practice, full variance-covariance matrices are rarely available with ERP series, and providing the full variance-covariance matrices would be nearly as difficult as providing the more desirable normal equations. In some of the early experiments, simple means were also taken of the ERP series to obtain one final series as well.

This method is meant to correspond to the BIH and USNO combined solutions (see section 1.1) as a comparison with the other solutions obtained in the simulation experiments. The correspondence is not really a very strict one, since:

(1) The BIH and USNO convert all of their ERP series to series with the same epochs via smoothing and filtering of the data. In this study, since the ERP series to be combined are all being created by the same software, the epochs are simply specified as the same for each series, thereby bypassing the need for (and the errors introduced by) such smoothing and filtering.

(2) No attempt is made here to eliminate the biases in any of the ERP series before taking the weighted means. These biases have been determined here, but only because the true ERP values are available. With real data the biases can never be found exactly, although attempts are made to recover them in the BIH and USNO solutions.

(3) The BIH and USNO solutions are relatively complex, depending greatly on the software, etc. used. It would be difficult to duplicate their methods exactly without lengthy study and programming, and/or obtaining copies of their software, which of course would not greatly contribute to the goals of this study.

However, trying to imitate the BIH and/or USNO methods carefully, in order to obtain a better reference comparison, would be a possible goal for future work, especially if real data was in use, and the absolute accuracies rather than the relative accuracies of these methods were being determined.

4.1.4.2 <u>Solution With All Observations ("Grand Solution") to Obtain ERP</u>. Once again, this method is well described by its title. One simply takes all the observations from all systems and does one "grand" adjustment to obtain a single ERP time series. There are some advantages to this method of obtaining ERP values; however an important disadvantage for this study was that no software yet exists which can directly handle laser ranging and VLBI data in a single adjustment. In fact, to properly consider this method, a software "trick" was performed, by iterating through the GEODYN and SOLVE programs (including manually updating the approximate values), which in this case simply set up and solved repeatedly the normal equations. The results of this method should be the same as if one massive (iterative) adjustment was performed.

4.1.4.3 <u>Combination of Normal Equations to Obtain ERP</u>. Directly combining normal equations resulting from several sets of data is not new. This method was described and used extensively here in the adjustment of optical satellite tracking data in the late 1960's and early '70's. See, for example, [Krakiwsky and Pope, 1968, pp. 61-62; Reilly et al., 1972, pp. 20-22, 58-59]. The primary use of this method has been to add the effect of new observations to a previously generated set of normal equations. Especially if the new set of observations is small, a large savings in computation time may result. Other advantages are that the effect of observations can also be removed from the normal equations, weights can easily be changed on the parameters, and constraints can be added to or subtracted from the normal equations, all without doing the substantial computation involved in setting up the normal equations again. In this particular application, however, we are combining normal equations obtained from inherently different types of observations, i.e., LLR, SLR, and VLBI.

The principle behind the addition of normal equations is quite simple, once some conditions are fulfilled:

(1) The parameters to be solved for must correspond exactly. For example, the time periods covered by the ERP must be identical, and the models and constants used completely compatible.

(2) The a priori variance of unit weight for each set of normal equations must be the same, or the normal equations converted so that they are the same.

(3) Any constraints or weights on the parameters already in any of the individual sets of normal equations must be removed. Constraints or weights on the parameters may then be added onto the final combined set of normal equations.

(4) The same approximate values must be used to generate each set of normal equations.

All of the above conditions have been fulfilled in all of the experiments done in this study (and no constraint equations were used).

Once these conditions are met, the normal equations may be simply combined by adding algebraically each corresponding element of the sets of equations, to get the elements of the final combined equations. For example, for each of the i observational systems, we have the normal equations:

$$N_i \; X_i = U_i \tag{14}$$

where N_i is the normal equation coefficient matrix, U_i is the normal equation constant vector, and X_i is the vector of corrections to the approximate values of the parameters X_i^o. We also have P_i, the weight matrix for the observations; L_i, the discrepancy vector of the observations and their values as computed via the mathematical model and approximate values X_i^o, and n_i the number of observations. Then, combining the normal equations of the three systems, we have

$$(N_1 + N_2 + N_3) \; X = (U_1 + U_2 + U_3) \tag{15}$$

or

$$N \; X = U \tag{16}$$

However, in this application, the normal equations being combined do not have precisely the same number of parameters (since additional parameters, such as for Lageos' and the Moon's orbit will exist), and in this case these parameters will each have their own row and column in the final combined normal matrix. Therefore the additions done above are not strict matrix additions, but must be done by corresponding rows and columns for the N_i matrices, and elements for the U_i vectors. Each unique parameter to be solved for will have its own row and column in the final N matrix, and its own element in the final X and U vectors. Once the combined matrix is formed, the parameters may (and are here) weighted by adding to the diagonal elements of the N matrix corresponding to the parameters being considered. Likewise, constraints, if any (and there are none used here), can be added onto the normal equations. Finally, we can compute the desired final values of the adjustment:

$$X = N^{-1} U \tag{17}$$

$$X^a = X + X^o \tag{18}$$

where X^a are the final adjusted values of the parameters and X^o is a vector of the unique approximate values of the $X_i^{\,o}$. The a posteriori variance of unit weight is given by:

$$\hat{\sigma}_0^2 = \frac{L_1^T P_1 L_1 + L_2^T P_2 L_2 + L_3^T P_3 L_3 + X^T U}{(n_1 + n_2 + n_3)} \tag{19}$$

and the final variance-covariance matrix for the parameters is of course:

$$\Sigma X_a = \hat{\sigma}_0^2 N^{-1} \tag{20}$$

4.2 Software Available for Simulation and Adjustment

In this section, brief descriptions are given of the software used in this study. First, the GEODYN and SOLVE programs are considered, with the reasons they were chosen for use given, along with a general description. The software used in the simulation of the VLBI data is discussed separately, and finally the several miscellaneous programs written and used for various purposes in this study are briefly described.

4.2.1 GEODYN and SOLVE: Adjustment and Primary Simulation Software

4.2.1.1 Selecting the Primary Software. Once the decision was made to undertake a simulation study, the important decision of how to obtain software was considered. The two obvious choices were to either write it, or get it from somewhere else. When this study was begun it was not clear how complex the simulation models would need to be, and since it was possible that real data might eventually be used, it was decided to obtain operational software from an outside source if possible. The tremendous complexity of any one of the observational systems (LLR, SLR, VLBI) precluded writing software for complex simulations or handling of real data. Hindsight would now indicate that most of the simulations performed could have been done with much simpler software. On the other hand the step to more complex simulations or real data handling would now be much easier than it would have been had that option been taken.

Once it was decided to go to an outside source for software, only a few obvious choices presented themselves:

1. GEODYN and its support programs, developed by GSFC, primarily with the help of EG&G-WASC.

2. PEP (the Planetary Ephemeris Program) at MIT.

3. GTDS (Goddard Trajectory Determination System), also developed by GSFC.

Other programs also existed which were quite capable of simulating and/or adjusting SLR or VLBI data, but not both types of data. These included various programs written here, at MIT, at GSFC, at JPL, at NGS, and at UTX. Since it was considered an extremely difficult task to modify these programs to process the other type of data, this option was rejected (unless the more general programs could also for some reason not be used).

In the end, the choice was made to go with GEODYN since:

(1) A version of GEODYN was already in operation here, proving it could be used.

(2) It was possible to reach several individuals who were very knowledgeable about the program, and in fact who were continuing to develop and improve it.

(3) The program was well tested for SLR to Lageos data simulation and adjustment. It also supposedly worked for VLBI data adjustment [Ma, 1983] and might be modified to work (again) for LLR data processing [Zelensky, 1984].

(4) A support program, SOLVE, was available to combine and solve normal equations generated by the GEODYN program. This option was considered useful as it would allow for easy combination solutions of various systems' data (and would best represent how such solutions would probably be done in practice with real data).

Neither PEP nor GTDS had these types of advantages. It is possible that one or the other would have easily processed LLR, SLR, and VLBI data, but since these were older, mostly now unsupported programs, there was no guarantee of this. In any case operation was not as likely to be as easy, the models in these programs were likely overly general and on the other hand probably outdated in terms of handling real data, and no one was clearly available to ask questions of.

4.2.1.2 <u>GEODYN and SOLVE Description</u>. No attempt will be made here to fully describe the GEODYN and SOLVE software. It should suffice here to say that GEODYN was developed as a general purpose program to allow determining and predicting satellite orbits, and estimating geodetic parameters. In recent years, it has been used primarily for the adjustment of SLR to Lageos data, but has working options to use, e.g., altimeter, satellite-to-satellite tracking, Doppler satellite tracking, and now VLBI data. Details concerning the program's theory are given in [Martin et al., 1976; Putney, 1977; Ma and Zelensky, 1982]. The only up-to-date documentation of the program is the operational instructions in [Eddy et al., 1983]. Incomplete documentation of the theory used for the SOLVE program also exists in [Estes, 1983] and for its operation in [Estes and Wildenhain, 1984].

In any case, all of the models previously discussed in section 4.1 are available in the GEODYN program, however, in most cases considerably more extended so that real data of various types may be preprocessed and adjusted.

The SOLVE program, in contrast to GEODYN, has basically just two simple functions: to combine normal equations, and to solve such combined normal equations. The input normal equations are generated in a GEODYN run using the "EMATRIX" option, or by the user, but in any case are in a format readable by SOLVE [Eddy et al., 1983, pp. C-42 to C-71]. The theory of combining normal equations was generally described here in section 4.1.4.3.

The 8210.7 version of GEODYN and the 8202.3 version of SOLVE are the specific versions of the programs which were used. Information on how these programs were obtained and problems encountered is given in the several semiannual reports on this work [DGSS, 1983-1986].

4.2.2 VLBI Simulation Software

One of the primary problems with using the GEODYN software was the lack of options in that program to simulate VLBI data, even though it was capable of processing real VLBI data. To overcome this problem it would be necessary to either modify GEODYN or to write/obtain other software to specifically simulate the VLBI data for use in GEODYN. The option to modify GEODYN itself was not very seriously considered, due to a) the complexity of the software, b) the lack of a suitable compiler here, and c) the lack of a clear way to allow GEODYN to access a VLBI schedule. After-the-fact discussion [Zelensky, 1986] also indicates that allocating the further needed array space in GEODYN would have also been quite difficult.

It was decided instead to write software capable of using a VLBI schedule as input, and which would generate a list of the observations to be simulated. A locally written program (VIP) could then be used to actually simulate the VLBI observations. In theory, this procedure sounded quite simple, but in practice it was found that matching the geometric models (e.g., precession, nutation) to those of GEODYN would be a very difficult task. Instead, an "iterative" procedure was adopted, where the VIP program simulated data that was then processed by GEODYN, and the residuals (resulting from the model differences) used to correct the simulated data (so that it now matched the "computed" values of GEODYN, i.e., it matched the models in GEODYN perfectly).

A further simple program was written to add noise to these observations as desired. A short description of each of these programs follows.

4.2.2.1 SKEDVIP: Reading a VLBI Schedule. A program called SKEDVIP was written to a) read a VLBI schedule in the standard Mark-III SKED program format [Vandenberg and Shaffer, 1983], b) shift this schedule by an integer number of sidereal days to a desired time period, c) duplicate observations from a single day over as many days as desired, and d) output all data in a format compatible with the VIP program (including one file each for station positions, source positions, and observing schedule).

4.2.2.2 VIP: Simulating the VLBI Data. The VIP program [Bock, 1980] was previously written here for the purpose of conducting interactive VLBI simulation experiments. A later (and improved) version of this program was still available, and with some suggestions from its author [Bock, 1984, 1985] was modified further. In particular, the program had a few errors corrected, was modified slightly to read an extended simulation schedule file (as created by SKEDVIP), to output simulated observations in GEODYN format, and to run as a batch (instead of TSO) program. In addition, precession (Newcomb's) and nutation (Woolard's) options were added so that the observations better matched GEODYN's "computed" observation values. Other changes are also listed in [DGSS, 1985, pp. 4-5].

4.2.2.3 VLBISIM: Final VLBI Data Simulation Program. As previously described, the simulated VLBI observations generated by the VIP program were not matching the GEODYN "computed" values very well. Rather than overhauling the VIP program so that it could match GEODYN's geometric models perfectly, a different procedure was followed. A program "VLBISIM" was written to use the residuals (i.e., the "O-C" values) from a GEODYN run with VIP simulated data, to correct the VIP data so that it matched GEODYN's models perfectly. The residuals are actually obtained from a file containing the "printout" of the GEODYN residual summary, as the values GEODYN outputs to a file are in single precision and are not precise enough in this application. Also as previously described (in 4.1.1.3) an IRCC routine "NORM02" is used by subroutine "NOISE" in VLBISIM to add random noise onto the simulated observations if desired. The final output simulated data can then be used in GEODYN as desired.

Hindsight shows that this entire VLBI data generation could be further simplified by dropping the use of the VIP program entirely. The SKEDVIP program could be modified to generate a GEODYN binary data file, with zero valued observations. The residuals in a GEODYN run with this data would then be (the negative of) the observations themselves. VLBISIM could still be used to read the residuals off the "printout" and add noise if desired.

4.2.3 Other Support Software

Several other programs were written in support of the simulation experiments. These are quickly described as follows:

(1) ERPSIM - This program is used to generate the ERP to be input to the simulations. Features of the program include: accepting any ERP series as a reference ERP series (in a quasi-BIH format), plotting any ERP series (either one parameter type at a time or X vs. Y polar motion), creating a new ERP

series using superimposed curves (as was described in detail in section 3.4), outputting the new series in either GEODYN "POLEUT" or VIP's format, outputting "POLEUT" approximate values for GEODYN use, and outputting ERP series averaged from the newly created one.

(2) PLOTERP - This program summarizes the results of the simulation experiments. As input it accepts: a) GEODYN "POLEUT" cards (simulation input ERP data or approximate values), b) the ERP "printout" of a GEODYN data reduction run, and c) the ERP (generalized format) "printout" from a SOLVE run. As output, it has options to list the input ERP series in a common format, and to plot the ERP values. It can also generate the weighted mean (or mean) ERP sets (described in 4.1.4.1) and similarly list or plot them. Multiple curves can be overplotted, and differences from a specified curve can be listed and/or plotted. Tables of the RMS and averages of these differenced curves are also computed. Most of the results of Chapter 5 were obtained via this program.

(3) PLACOOR - This is a modified version of a program by E. Pavlis, which in turn used routines provided by E. Standish of JPL. It can read any JPL Planetary and Lunar Ephemeris, and compute and print the state vector of any solar system object (in the ephemeris). It was used to obtain the lunar state vector given in Table 9.

(4) PLOTSTAT - This program accepts as input station numbers, abbreviations, and positions in "STAPOS" GEODYN format (optionally modified to contain station names). Maps showing the station positions, at any desired scale, of any section of the world, with any (available) projection can then be plotted. The Shore Outline Plotting Package (SOPP) routines have been used within it [Krieg and Archinal, 1986]. Additionally, the stations are listed with their Cartesian geocentric and geodetic coordinates, using a specified ellipsoid. The maps shown in Chapter 3 were created with this program.

5. RESULTS

Now that we have discussed the reasons for doing simulation experiments, as well as the input to those simulations and the models for them, the results can finally be presented. These "results" consist primarily of the differences between the ERP recovered in the simulation experiments, and their "actual" values (input to the simulations), as well as comparisons of the recovered ERP variances and correlations with those differences and between the various methods. As previously indicated these recovered ERP are obtained from LLR, SLR to Lageos, VLBI, weighted mean (of the preceding), normal equation combination, and grand solutions.

In effect, by lookng at how well the ERP are recovered, we will be comparing the **accuracies** of these methods, while by looking at the standard deviations and correlations, we will be comparing the **precision** of these methods. Ideally, we may only be interested in the accuracies. However, this cannot be the only factor considered since: a) the accuracy will be dependent on the specific observations being adjusted (although this effect has been minimized by considering a large number of observations), and b) the recovered parameters are not complete without their statistics, as it is this complete set of information which will (or should) be used in further applications.

The comparisons are presented in both graphical and statistical form. For the graphical presentations, for each experiment three plots are given, one each for the x_p, y_p, and UT1-UTC parameters. Some of the plots show both the "actual" values and the recovered ERP so that the fluctuations of the ERP can be displayed along with their recovered values. However, the majority of the plots show only the differences of the recovered ERP from the input "actual" ERP, thus allowing the variations of the recovered values to be more easily seen. In all cases (except the "actual" value case) additional symbols above and below a plotted symbol indicate the one standard deviation range for the ERP value ("error bars" or "error bands"). In the case of the few plots which show the approximate values used, these symbols represent the standard deviations used to determine the weights for the parameters. Two special points should be made concerning these graphs. First, **ideally these curves should be plotted as step functions,** since the ERP values are averaged over a time interval, not values at a specific epoch. In the interest of clarity on the plots, only the midpoint of each step was plotted, with connecting lines to indicate the trend of each curve, and to better indicate the position of a curve in the often crowded areas on the plots. Secondly, some attempt has been made to make the scales of the plots uniform. However, due to the variations and range of the parameters shown, this uniformity is not very general and the scales should be examined when comparing plots, for example, for different ERP recovery periods, or different total time periods. The abscissa is always given in days of time from the start of the simulation, and is either four or 15 days duration in length. The ordinate is in arcseconds or milliarcseconds for polar motion, and seconds or milliseconds for UT1-UTC, the milli- units being used when differences from the "actual" values are being plotted.

Tables of statistics are also given for each simulation experiment, and it is perhaps through these tables that most of the important results can be gleaned. For each experiment (i.e., each ERP recovery period, of six hours, 12 hours, etc.), a separate table shows the results for x_p, y_p, and UTC-UTC. For each method and parameter type, the RMS, average, and maximum difference of the recovered time series from the "actual" time series is given, along with the average standard deviation for that time series. The RMS differences can be used to indicate the accuracy or relative accuracy of a given method if no biases (constant offsets of an ERP series from the "actual" time series) exist, the average differences are in fact the values of any biases if they exist, and the maximum differences indicate the "worst case" possible for a given method.

The average standard deviations are also given to provide another check on the RMS differences, since when enough observations have been used, one would expect these values to be nearly the same. The average standard deviations also provide another type of measure as to which solution is best. See 5.1.6 for information on the average standard deviation over all three ERP components, and on the correlations between parameters.

Also for each of these values, an additional number is given in parentheses which is the (absolute value) multiple of that value of the lowest value for all the methods. While the size of the statistics themselves give the reader an idea of the overall accuracy of each method, the additional numbers allow comparisons of the relative accuracy of each method. For example, the method with a "1.0" was the most accurate for that ERP component and period, a method with a multiple between "1.0" and (say) "3.0" is doing quite well, while large numbers ("10" or more) indicate the recovery was much poorer than the other methods (either because the method is not as strong due to geometry or lack of data, or because biases exist in the ERP values).

It is difficult to define in some cases whether one method or another is "best." It will be obvious that in many cases, the various methods give very similar results, both by their plotted curves, RMS, etc. differences, and average standard deviations. In these cases it is of course difficult to come to any important conclusions. However, as will be seen, certain of the experiments clearly indicate that one method or another is of the highest accuracy, depending on the ERP recovery period, and the parameter of interest. Special emphasis will of course be given to these cases, especially in the conclusions of the next chapter.

5.1 Fifteen-Day Duration Simulation Results

In this section, the most important results of this study are presented, concerning the recovery of ERP from 15 days of simulation. These results are presented in six subsections, the first five containing results for the recovery of ERP with periods of six hours, 12 hours, one day, two days, and five days respectively. The last subsection contains information on the standard deviations and correlations of the ERP recovery solutions.

As previously indicated, these results are shown in the form of a) plots of the recovered curves (for the six-hour ERP period only), b) plots of the differences of these curves from their correct values, and c) tables of statistics. Discussions concerning these results are then presented in approximately the same order, noting the most accurate methods found (overall, and for polar motion and UT1-UTC individually), the methods of intermediate accuracy, and the methods of lowest accuracy or with biases. Most of these discussions, following the overall purpose of this study, concern only the **relative** accuracies of various methods. However, since these simulations also indicate the best possible accuracy obtainable by these methods, the actual magnitudes are also discussed.

For all of the solutions with results presented here (except for weighted mean and mean solutions), the final a posteriori variances of unit weight are presented in the Appendix.

5.1.1 Six-Hour ERP Results

The results for the six-hour period ERP are considered here, and their importance should be obvious considering the continuing interest in obtaining high time resolution ERP values. Plots of the first four days (for clarity) of the recovered curves themselves are shown in Fig. 6, and of their differences from the correct values (the "actual" or simulation input values) in Fig. 7. The statistics of these differences are given in Table 13.

Fig. 6 shows very little of note, except the clear tracking by all of the methods of the correct ("ACTUAL") ERP, with the exception of the LLR solution, which has noticable "error bars", and poorer results near the start and (not shown here) stop times. This is apparently due to the poor geometric strength of the LLR network over six-hour periods, and especially over what are actually three-hour periods at the endpoints. The three horizontal lines of circles represent the approximate values and their standard deviations (used to determine their weights), which were the same (in polar motion and UTC-UTC) for all but some test solutions. This indicates that as long as any reasonable approximate values are used, all of these methods, even LLR to a large extent, are capable of nearly correctly recovering the ERP values.

One important conclusion that can be drawn from these solutions is from their very existence. They demonstrate that (under the assumption of very high data rates) six-hour ERP can be recovered from at least SLR and VLBI observations alone, and to a lesser accuracy from LLR observations. The solutions at the beginning and end of the ERP time series, actually being for only three-hour ERP, indicate that solutions over such shorter periods are also possible. Although VLBI solutions for ERP over such periods have been reported in the literature [Carter and Robertson, 1985b; Robertson et al., 1985], solutions over such periods have apparently not been reported for LLR and SLR.

Fig. 7 shows plots of the differences of the recovered ERP time series from their true values. Because of its large variations, the LLR results are not plotted. We now see the variation in the VLBI and SLR results for polar motion quite well. Two points of note are that a) the SLR variations seem greater than the VLBI, and b) both the SLR and VLBI results appear to be

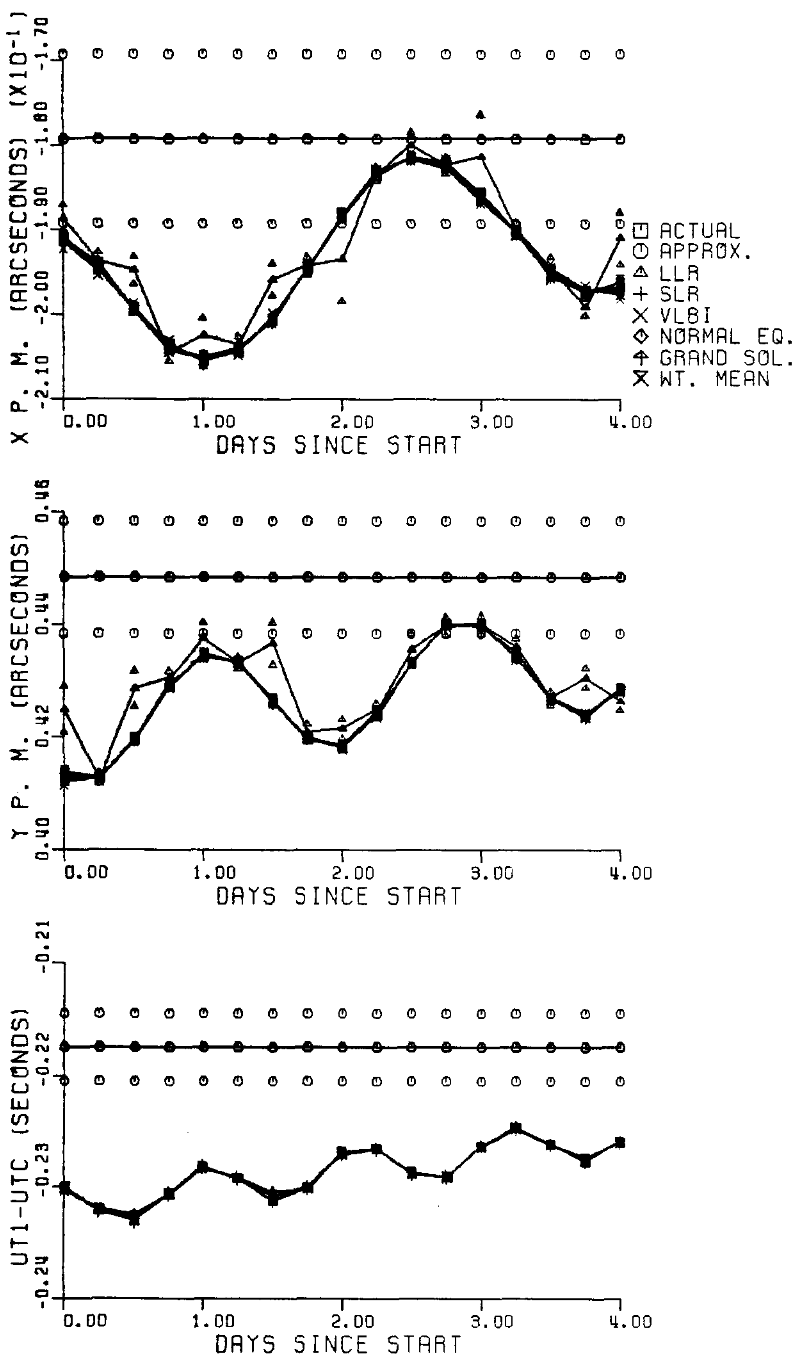

Fig. 6 Six-hour ERP time series recovered from four days of simulated data.

Symbols for each curve are connected by lines for clarity. Unconnected symbols above and below represent error bars (or standard deviations used for weights). "ACTUAL" - simulation input. "NORMAL EQ." - Results of normal equation combination solution. "GRAND SOL." - Results of grand solution. "WT. MEAN" - Results of wt. mean solution.

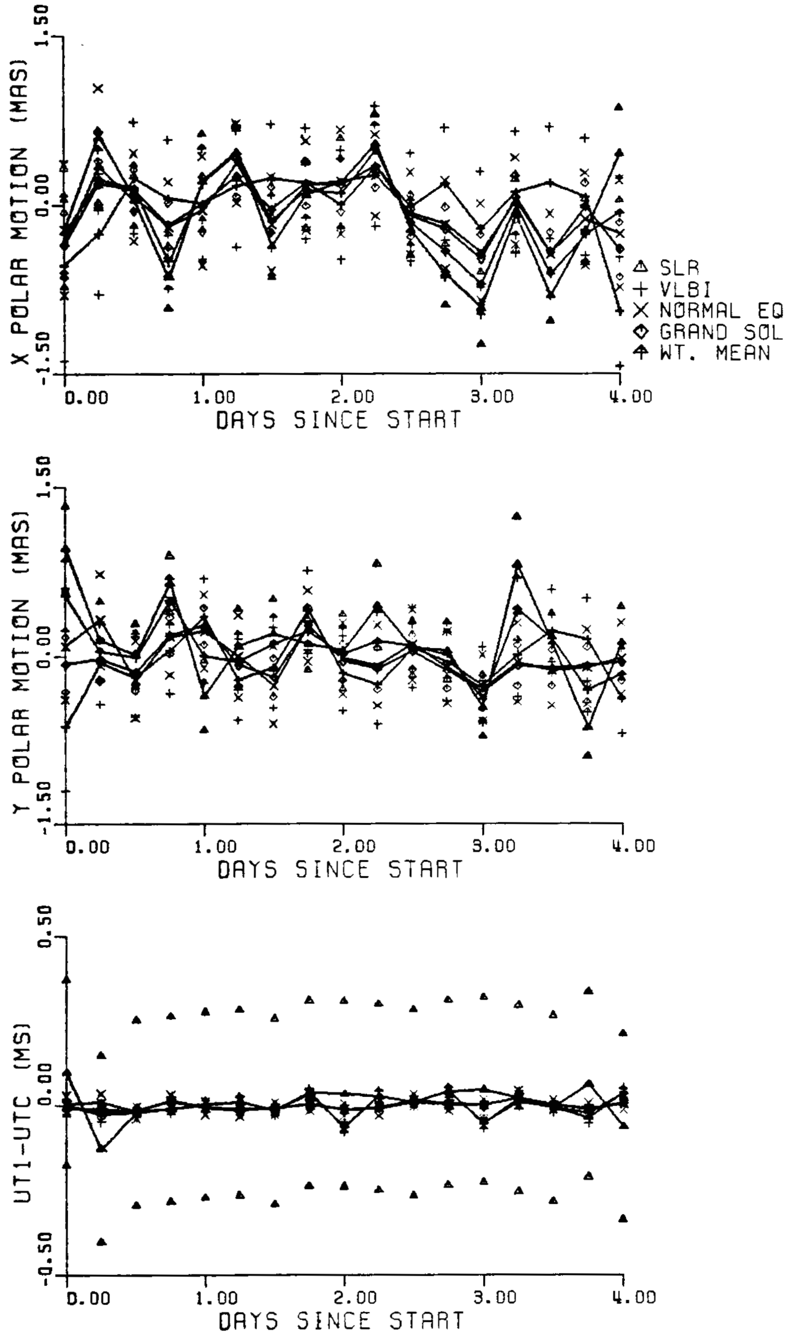

Fig. 7 Six-hour ERP differences, four days of simulated data, LLR not shown.

Differences are from true values (simulation input). Symbols for each time series are connected by lines for clarity. Unconnected symbols above and below represent error bars. "NORMAL EQ." - Results of normal equation combination solution. "GRAND SOL." - Results of grand solution. "WT. MEAN" - Results of wt. mean solution.

Table 13 Statistical Differences, Six-Hour ERP Recovery

All differences are from simulation input over 61 ERP recovery periods.

X POLAR MOTION (MAS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S.D.*
LLR	1.8(7.8)	0.3(551.)	5.7(11.6)	1.4(7.0)
SLR	0.4(1.5)	0.0(37.0)	-0.9(1.9)	0.3(1.4)
VLBI	0.5(2.2)	0.1(133.)	1.6(3.2)	0.5(2.5)
NORMAL EQ.	0.2(1.0)	0.0(53.3)	0.6(1.2)	0.4(2.0)
GRAND SOL.	0.2(1.0)	0.0(64.7)	-0.5(1.0)	0.2(1.0)
WT. MEAN	0.3(1.2)	0.0(1.0)	-0.7(1.4)	0.2(1.2)
MEAN	0.6(2.8)	0.1(215.)	1.7(3.5)	

Y POLAR MOTION (MAS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S.D.*
LLR	3.0(17.2)	0.7(211.)	12.3(22.6)	1.6(9.6)
SLR	0.3(1.8)	0.0(12.4)	1.0(1.8)	0.3(1.7)
VLBI	0.4(2.1)	0.0(8.8)	1.1(2.0)	0.4(2.2)
NORMAL EQ.	0.2(1.0)	0.0(1.3)	-0.6(1.2)	0.3(2.0)
GRAND SOL.	0.2(1.0)	0.0(1.0)	-0.7(1.2)	0.2(1.0)
WT. MEAN	0.2(1.2)	0.0(10.3)	0.5(1.0)	0.2(1.3)
MEAN	1.0(5.8)	0.2(77.3)	4.2(7.7)	

UT1-UTC (MS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S.D.*
LLR	0.14(13.4)	0.01(17.2)	0.68(24.5)	0.08(8.7)
SLR	0.04(3.9)	0.00(1.0)	-0.13(4.6)	0.27(28.4)
VLBI	0.03(2.9)	0.00(3.0)	-0.07(2.4)	0.02(1.7)
NORMAL EQ.	0.01(1.0)	0.00(1.2)	0.03(1.0)	0.02(2.0)
GRAND SOL.	0.01(1.1)	0.00(1.7)	0.03(1.2)	0.01(1.0)
WT. MEAN	0.03(2.7)	0.00(2.9)	-0.06(2.3)	0.02(1.6)
MEAN	0.05(4.4)	0.01(6.4)	0.22(7.8)	

Values in parentheses show (absolute value) multiples of lowest value in column.

Notes: 15 days of simulated data.
"MEAN" is mean of LLR, SLR, and VLBI values.

oscillating, probably due to the variation in the original simulation input ERP. The weighted mean, the normal equation combination, and the grand solutions all appear to be recovering the ERP slightly better than the SLR and VLBI solutions. For UT1-UTC, the "error bar" symbols for SLR are an overriding feature of the plot, and although the SLR UT1-UTC is obviously worse than that of the other methods, the "error bars" are far too large. This is probably due to the relatively loose (one meter) weights put onto the Lageos orbit, which propagates into these large "error bars." A close examination of the plot also shows detectable "error bars" for VLBI, but generally the VLBI, weighted mean, normal equation combination, and grand solutions all look of similar accuracy.

Suspecting that the LLR data may be degrading the combination solutions, additional such solutions were done without any contribution from the LLR data at all. Fig. 8 shows a plot of these results, in the same way as Fig. 7, but now with no LLR contribution at all. We can see almost no difference between the figures, demonstrating that any such degradation is unlikely.

Table 13 shows the statistics of these difference curves, with Table 14 showing the results without any influence from LLR. One of the most significant results is that the normal equation combination and grand solutions **always** give the lowest RMS values for polar motion and UT1-UTC, **although** the weighted mean, SLR, and VLBI results are only slightly poorer. The normal equation combination and grand solutions give results about two to four times better than the SLR and VLBI results. The weighted mean results are also only slightly worse for polar motion. Looking at the average differences shows that only the LLR and mean X and Y, and VLBI X polar motion solutions have any noticeable biases. The relative maximum difference results are similar to the RMS difference results. For polar motion, SLR gives slightly better but comparable results to VLBI. For UT1-UTC, SLR gives slightly worse RMS results than VLBI, and all of the methods except LLR and the mean have negligible biases. This is surprising considering that SLR solutions usually have some bias in their UT1-UTC determinations, since UT1-UTC is very highly correlated with the satellite orbit orientation. The lack of this bias in the SLR results is probably due to several factors, including: a) the lack of any consideration of the systematic errors known to exist in the force models for Lageos' orbit, b) the use of approximate values for the satellite orbit parameters that are the same as their simulation input values, and c) the recovery of short-period UT1-UTC, allowing for slightly better separation (lower correlation) of these parameters from orbit parameters ([Eanes, 1986], also see 5.1.6). Additionally, the mean (of the LLR, SLR, and VLBI) results are also quite poor, mostly due to the LLR results. This strong effect on the mean by a system which has bad data was felt reason enough to not consider (unweighted) mean solutions further, especially when standard deviations are almost always available with ERP solutions.

As to the average standard deviations, we see results that are similar to the RMS and maximum difference results. As expected (if only because they contain the most observations), the grand and then the normal equation combination and weighted mean solutions have the smallest standard deviations. The VLBI, and the SLR polar motion values are only slightly worse, but the LLR, and SLR UT1-UTC (especially) values are quite large in comparison.

Next we can look at the absolute accuracies obtained. For polar motion we see that the values obtained from the simulations are in the ranges of 0.4 to 0.5 mas for SLR or VLBI and 0.2 to 0.3 mas for the weighted mean, normal equation combination, and grand solutions. For UT1-UTC, SLR and VLBI give values of 0.04 and 0.03 ms respectively. For the weighted mean and data combination solutions we obtain by simulation 0.03 and 0.01 ms. For LLR, we obtain for UT1-UTC a poorer RMS of 0.14 ms.

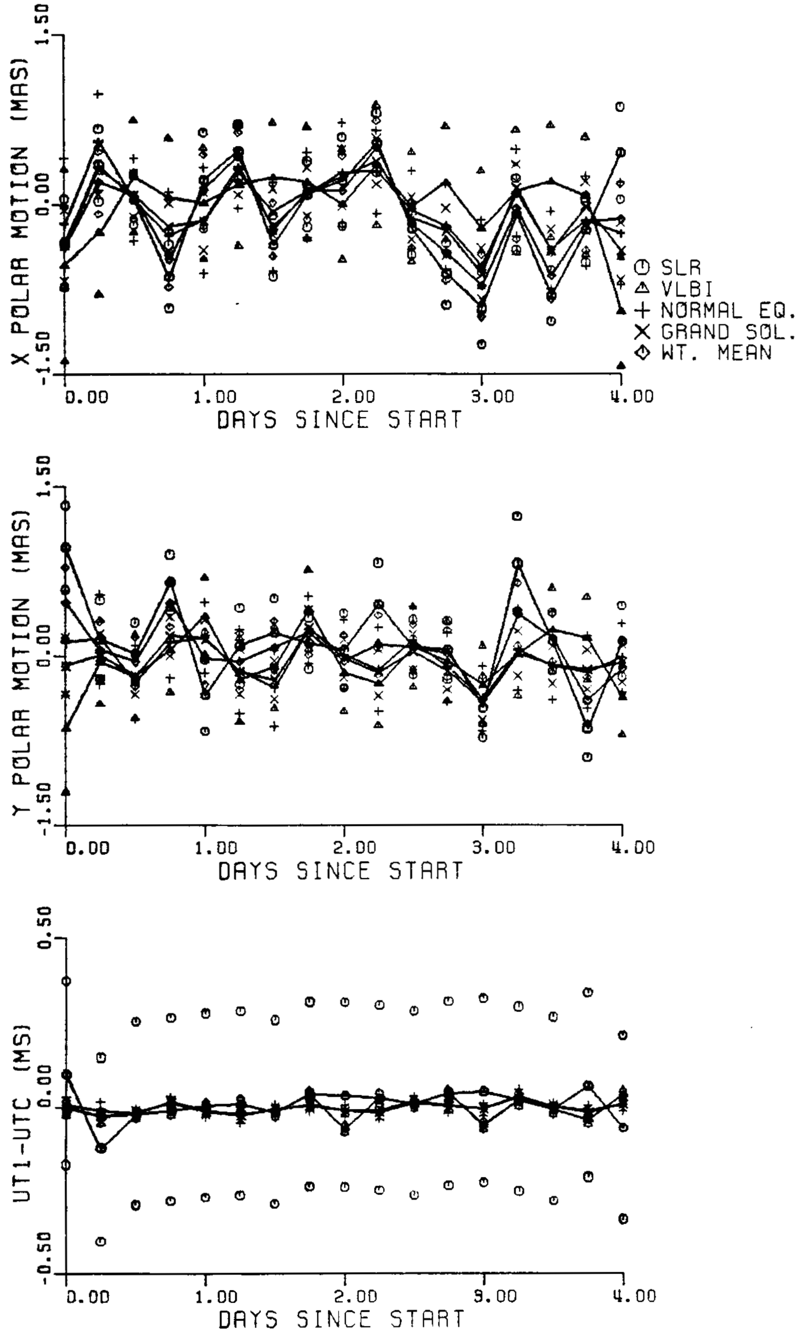

Fig. 8 Six-hour ERP differences without effects of LLR, four days of simulated data.

Differences are from true values (simulation input). Symbols for each time series are connected by lines for clarity. Unconnected symbols above and below connected ones represent error bars. "NORMAL EQ." - Results of normal equation combination solution. "GRAND SOL." - Results of grand solution. "WT. MEAN" - Results of wt. mean solution. (The effect of LLR data is not included in any of the combination solutions.)

Table 14 Statistical Differences, Six-Hour ERP Recovery, No LLR

All differences are from simulation input over 61 ERP recovery periods.

X POLAR MOTION (MAS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S. D.*
SLR	0.4(1.6)	0.0(17.9)	-0.9(1.7)	0.3(1.3)
VLBI	0.5(2.2)	0.1(63.9)	1.6(2.9)	0.5(2.4)
NORMAL EQ.	0.2(1.0)	0.0(18.3)	0.6(1.0)	0.4(1.8)
GRAND SOL.	0.2(1.1)	0.0(28.9)	-0.6(1.1)	0.2(1.0)
WT. MEAN	0.3(1.2)	0.0(1.0)	-0.7(1.3)	0.2(1.2)

Y POLAR MOTION (MAS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S. D.*
SLR	0.3(1.9)	0.0(33.8)	1.0(2.0)	0.3(1.7)
VLBI	0.4(2.2)	0.0(24.0)	1.1(2.2)	0.4(2.1)
NORMAL EQ.	0.2(1.0)	0.0(1.7)	-0.5(1.0)	0.3(1.8)
GRAND SOL.	0.2(1.0)	0.0(1.0)	-0.5(1.1)	0.2(1.0)
WT. MEAN	0.2(1.2)	0.0(22.7)	-0.5(1.0)	0.2(1.3)

UT1-UTC (MS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S. D.*
SLR	0.04(3.5)	0.00(1.0)	-0.13(4.2)	0.27(26.9)
VLBI	0.03(2.6)	0.00(3.0)	-0.07(2.2)	0.02(1.6)
NORMAL EQ.	0.01(1.0)	0.00(1.5)	0.03(1.0)	0.02(1.8)
GRAND SOL.	0.01(1.0)	0.00(1.8)	0.03(1.1)	0.01(1.0)
WT. MEAN	0.03(2.6)	0.00(3.0)	-0.07(2.2)	0.02(1.6)

Values in parentheses show (absolute value) multiples of lowest value in column.

Notes: 15 days of simulated data.
"NORMAL EQ.", "GRAND SOL.," and "WT. MEAN" include only the effects of the SLR and VLBI normals and solutions respectively.

Finally, we note there are negligible differences in the combination solution statistics between Tables 13 and 14, just as previously noted in looking at the plots of the curves. This indicated further that all three of the combination solutions are not degraded by data of noticeably worse accuracy, at least for this six-hour ERP recovery, where no biases (large average differences) are present.

5.1.2 Twelve-Hour ERP Results

The differences of the ERP time series (over the first four days) from their correct values are shown plotted in Fig. 9, and the statistics of these time series are given in Table 15.

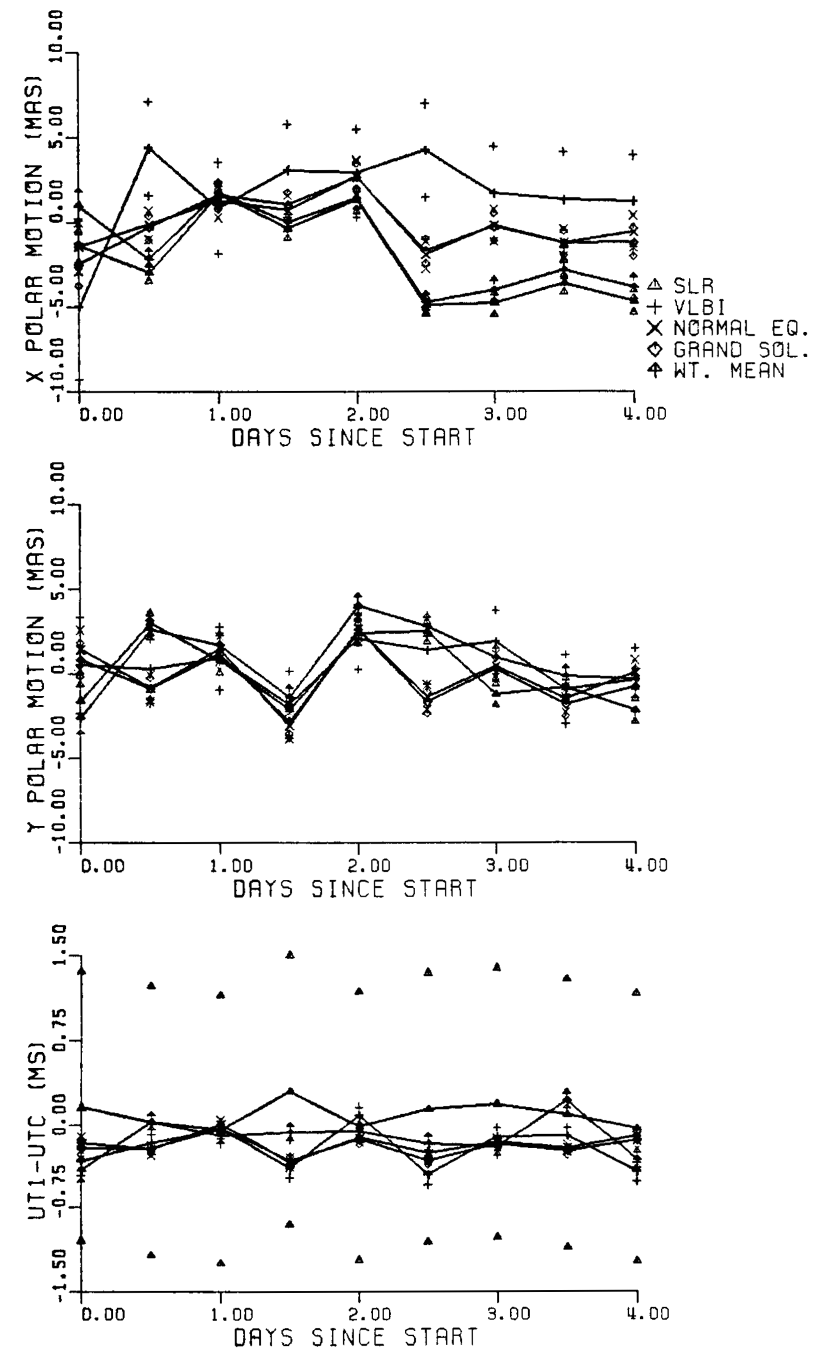

Fig. 9 Twelve-hour ERP differences, four days of simulated data. Differences are from true values (simulation input). Symbols for each time series are connected by lines for clarity. Unconnected symbols above and below connected ones represent error bars. "NORMAL EQ." - Results of normal equation combination solution. "GRAND SOL." - Results of grand solution. "WT. MEAN" - Results of wt. mean solution.

Table 15 Statistical Differences, 12-Hour ERP Recovery

All differences are from simulation input over 31 ERP recovery periods.

X POLAR MOTION (MAS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S. D.*
LLR	8.0(4.6)	-1.7(5.3)	19.8(4.4)	2.1(3.8)
SLR	3.8(2.2)	-0.7(2.2)	-10.3(2.3)	0.6(1.1)
VLBI	2.9(1.7)	0.7(2.1)	5.0(1.1)	2.8(5.0)
NORMAL EQ.	1.7(1.0)	-0.4(1.4)	-5.1(1.1)	0.9(1.6)
GRAND SOL.	1.7(1.0)	-0.3(1.0)	-4.5(1.0)	0.8(1.4)
WT. MEAN	3.4(1.9)	-0.8(2.5)	-9.7(2.1)	0.6(1.0)

Y POLAR MOTION (MAS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S. D.*
LLR	14.8(11.0)	1.8(24.3)	35.5(11.7)	2.1(3.7)
SLR	2.2(1.6)	-0.1(1.0)	-4.9(1.6)	0.6(1.1)
VLBI	1.4(1.0)	0.4(6.0)	4.2(1.4)	2.0(3.5)
NORMAL EQ.	1.5(1.1)	-0.3(3.5)	-3.3(1.1)	0.8(1.3)
GRAND SOL.	1.5(1.1)	-0.2(2.4)	-3.0(1.0)	0.7(1.2)
WT. MEAN	2.2(1.6)	-0.1(1.0)	-5.1(1.7)	0.6(1.0)

UT1-UTC (MS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S. D.*
LLR	0.43(2.1)	-0.04(1.0)	1.09(2.5)	0.13(3.4)
SLR	0.29(1.5)	0.25(6.0)	0.65(1.5)	1.20(31.9)
VLBI	0.29(1.5)	-0.04(1.0)	0.49(1.1)	0.09(2.3)
NORMAL EQ.	0.20(1.0)	-0.05(1.2)	-0.47(1.1)	0.04(1.2)
GRAND SOL.	0.21(1.0)	-0.05(1.2)	-0.45(1.0)	0.04(1.0)
WT. MEAN	0.22(1.1)	-0.05(1.3)	-0.47(1.1)	0.07(1.9)

Values in parentheses show (absolute value) multiples of lowest value in column.

Note: 15 days of simulated data.

Once again, the normal equation combination and grand solutions have very small differences from their correct values (the zero of the plots), and the SLR, VLBI, and weighted mean results are not quite as good, but still similar and excellent in their own right. The LLR results are still much poorer, especially still at the start and (not shown on figure) end points where there was actually less data in the LLR solutions, and further include clear oscillations of the results around the correct value. These latter oscillations are probably related to a similar oscillation of the input polar motion values. SLR continues to have "error bars" which are larger than the actual errors seen in its UT1-UTC solution.

As to Table 15, we see that the normal equation combination and grand solutions give the lowest RMS for X polar motion and UT1-UTC, but interestingly, now VLBI is giving the best result in Y polar motion. SLR gives results for polar motion now slightly worse than VLBI, but is giving similar results to VLBI for UT1-UTC. However, we see that the average difference for SLR in UT1-UTC is now becoming noticable (0.25 ms) showing a bias is occurring. As already shown, the LLR results are still much poorer than the other methods. However, the error relative to the other methods (parentheses numbers) is certainly decreasing, especially in UT1-UTC. A further point is that now all the methods are giving significant average differences (biases) in polar motion, except perhaps for the SLR and weighted mean Y solutions.

The average standard deviations, although still generally the same in their relative numbers, no longer match as well in absolute magnitude the RMS or maximum differences. For polar motion, the weighted mean has become the solution with the smallest average deviation, while the SLR, normal equation combination, and grand solutions follow closely. The LLR and VLBI solutions now have the largest average standard deviations. For UT1-UTC, the normal equation combination and grand solutions continue to give the smallest values. The LLR, VLBI, and weighted mean solutions give average standard deviations 1.9 to 3.4 times larger, while the much larger SLR value continues to reflect the possiblity of a large bias occurring (although still larger than the 0.25 ms average difference which actually resulted).

As to the absolute magnitude of the errors of these methods, we see that the SLR, VLBI, and weighted mean RMS differences have now jumped up to 1.4 to 3.8 mas. The normal equation combination and grand solutions are also up to the 1.5 mas level. As to UT1-UTC, the accuracies for SLR and VLBI have jumped substantially up to 0.3 ms. The combination solutions show a similar accuracy level at about 0.2 ms. For LLR, we obtain a UT1-UTC RMS difference of 0.4 ms.

5.1.3 One-Day ERP Results

We now turn to the recovery of ERP over one day periods. Fig. 10 shows the usual plots of the curve differences from their correct values, and Table 16 shows the statistics of these differences.

We now clearly see how well the normal equation combination and grand solution values track each other as well as their closeness to zero. LLR is still showing poorer relative polar motion values than the other methods, again with some oscillation seemingly connected with the input ERP. There is also some oscillation, but now LLR is giving one of the best curves for UT1-UTC. We also see that SLR continues to do satisfactorily for polar motion, but has now developed a large approximately 2 ms bias in UT1-UTC, barely included in its usual large "error bar" envelope. Its overall shape, however, is similar to that of the other methods, and fairly flat. VLBI is now giving what could be the best Y polar motion values, but X polar motion values with some oscillations. It should also be noted that for UT1-UTC, the weighted mean, normal equation combination, and grand solutions track the VLBI results quite closely. This is probably due to the relatively large standard deviations associated with the LLR and SLR solutions, thereby giving the VLBI data most of the weight in the combination solutions.

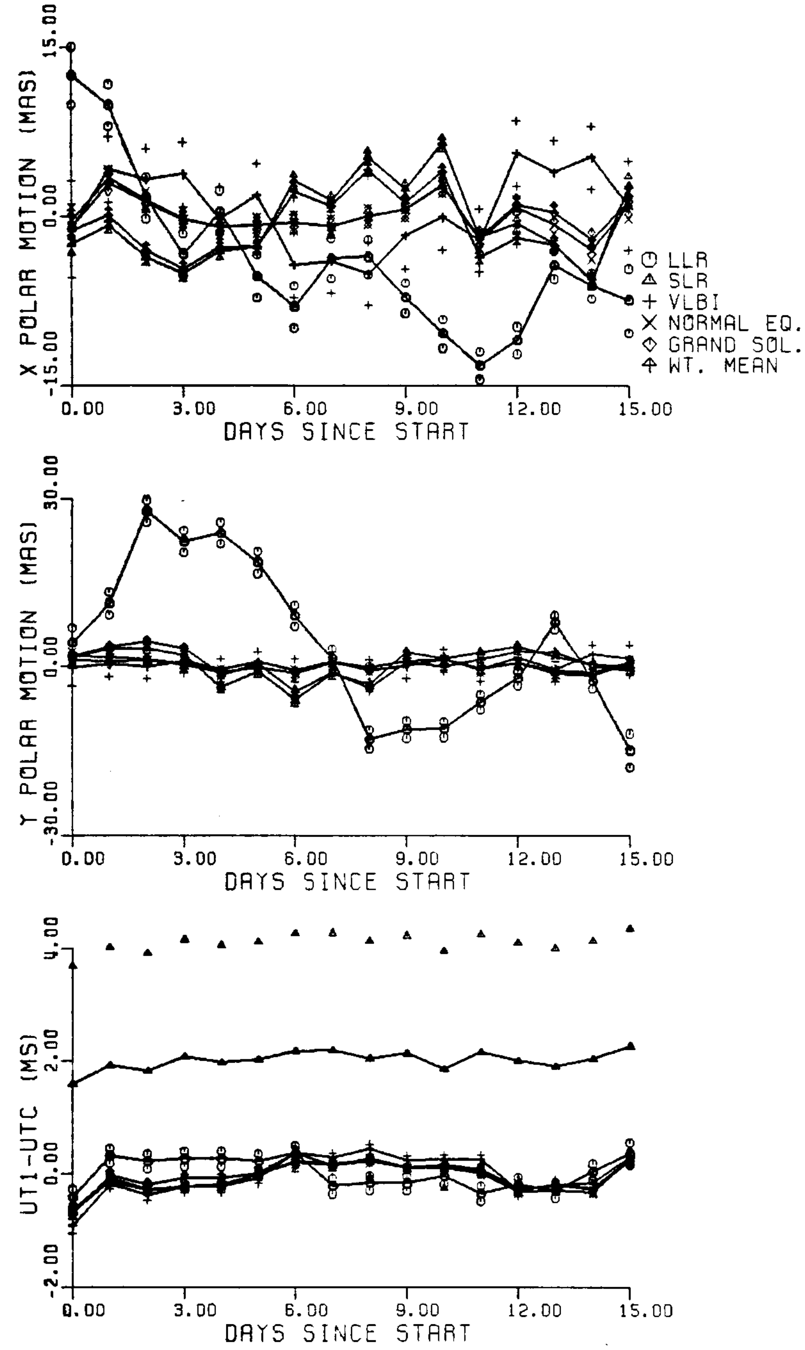

Fig. 10 One-day ERP differences, 15 days of simulated data. Differences are from true values (simulation input). Symbols for each time series are connected by lines for clarity. Unconnected symbols above and below connected ones represent error bars. "NORMAL EQ." - Results of normal equation combination solution. "GRAND SOL." - Results of grand solution. "WT. MEAN" - Results of wt. mean solution.

Table 16 Statistical Differences, One-Day ERP Recovery

All differences are from simulation input over 16 ERP recovery periods.

X POLAR MOTION (MAS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S. D.*
LLR	7.7(5.4)	-3.7(133.)	-13.2(4.5)	1.7(3.3)
SLR	3.6(2.5)	-0.5(16.1)	6.5(2.2)	0.6(1.1)
VLBI	3.4(2.4)	0.6(22.3)	5.6(1.9)	3.0(5.8)
NORMAL EQ.	1.6(1.1)	0.0(1.0)	3.4(1.2)	0.8(1.6)
GRAND SOL.	1.4(1.0)	0.1(4.4)	3.0(1.0)	0.8(1.5)
WT. MEAN	3.0(2.1)	-0.9(31.9)	-5.5(1.9)	0.5(1.0)

Y POLAR MOTION (MAS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S. D.*
LLR	14.1(16.5)	4.2(41.2)	27.8(21.2)	1.8(3.5)
SLR	2.8(3.3)	0.4(4.3)	-6.0(4.6)	0.6(1.1)
VLBI	0.9(1.1)	0.3(3.0)	2.1(1.6)	2.1(4.1)
NORMAL EQ.	1.1(1.3)	0.1(1.4)	1.9(1.5)	0.7(1.3)
GRAND SOL.	0.9(1.0)	0.1(1.0)	-1.3(1.0)	0.6(1.2)
WT. MEAN	2.6(3.0)	0.6(5.5)	4.7(3.6)	0.5(1.0)

UT1-UTC (MS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S. D.*
LLR	0.27(1.0)	0.01(1.0)	-0.42(1.0)	0.14(3.8)
SLR	2.01(7.9)	2.01(148.)	2.26(5.4)	2.10(57.4)
VLBI	0.35(1.4)	-0.04(3.2)	-0.92(2.2)	0.09(2.5)
NORMAL EQ.	0.26(1.0)	-0.08(5.9)	-0.64(1.5)	0.04(1.1)
GRAND SOL.	0.26(1.0)	-0.08(5.9)	-0.68(1.6)	0.04(1.0)
WT. MEAN	0.26(1.0)	-0.02(1.5)	-0.70(1.7)	0.08(2.1)

Values in parentheses show (absolute value) multiples of lowest value in column.

Note: 15 days of simulated data.

Table 16 shows that the grand solution has the best RMS for all three ERP components. The normal equation combination (X and Y) and VLBI Y results are only slightly worse for polar motion while the LLR, normal equation combination, and weighted mean solutions all give UT1-UTC effectively as good as the grand solution. The weighted mean and SLR solutions also provide results 2.5 to 3.3 times worse for polar motion. All the other solutions for polar motion show relatively large average differences (biases) in comparison to the normal equation combination and grand solutions, with the LLR polar motion now having quite large values. For UT1-UTC, all of the methods give similar results except for the SLR solution as it becomes much worse due to a large average difference (bias). The maximum difference values continue to reflect the RMS difference values.

The trend established with the 12-hour values for the average standard deviations continues. The weighted mean gives the lowest value for polar motion however, with the grand solution still giving the lowest value for UT1-UTC. The SLR, normal equation combination, and grand solutions give slightly higher values for polar motion (within a factor of 1.6), while LLR give values twice as large, and VLBI two to three times as large. For UT1-UTC, the normal equation combination solution value is almost as small as the grand solution value, and the LLR, VLBI, and weighted mean values are two to four times larger. The value of 2.1 ms for SLR matches the difference values (2.01, 2.01, and 2.21) extremely well, reflecting the existence of the large average difference (bias). Why are the grand and normal equation combination solution values no longer the lowest for polar motion? Apparently because the values for LLR (1.7, 1.8) are much lower than the LLR biases (-3.7, 4.2). Although this overweighting of the LLR influence should affect all of the combination solutions similarly, apparently the lack of consideration of the correlations "helps" the weighted mean solution in this case. This will be discussed further in 5.1.6.

The absolute magnitudes for polar motion generally continue the trends already seen, with the SLR and VLBI (at least for X) RMS values increasing slightly, now up to 2.8 to 3.6 mas. The LLR results are staying the same at about 8 and 14 mas, and the combination solutions are staying at between 1 and 1.6 mas. For UT1-UTC, LLR continues to increase in accuracy, now nearly to the currently quoted accuracy of 0.3 ms. The other methods have similar RMS's, except for SLR which now has the large 2 ms bias.

5.1.4 Two-Day ERP Results

We have now moved up to an ERP solution period where continuous real data solutions are common, at least with SLR and, e g., Doppler tracking of the NNSS. We now have over the 15 days of simulated data eight ERP periods (with the first and the last actually using only one day of data). The plot of the curve differences using this two-day recovery period is Fig. 11, while the statistics are given in Table 17.

Since the methods are now giving similar results, and with fewer data points, the plots (in Fig. 11) have become clearer. As usual, for polar motion the grand and normal equation combination solutions give the best results, with VLBI also giving Y polar motion just as well. SLR polar motion and VLBI X polar motion are 2.4 to 2.8 times worse than the others, and as usual LLR gives much poorer polar motion than all the other methods, although it is improving in Y. Some oscillations are still visible in the LLR results as well. For UT1-UTC, all of the methods except SLR are giving excellent results, with the LLR results perhaps the best. As usual, there is a bias in the SLR results (now 4 ms) but the shape of the curve is fairly flat (correct).

As to Table 17, the grand solution gives the best polar motion, and now LLR is clearly giving the best UT1-UTC. The RMS of LLR for UT1-UTC is 1.5 to 1.8 times smaller than any other method, and its bias is negligible in comparison to the other methods. Our other comments on Fig. 11 are also confirmed.

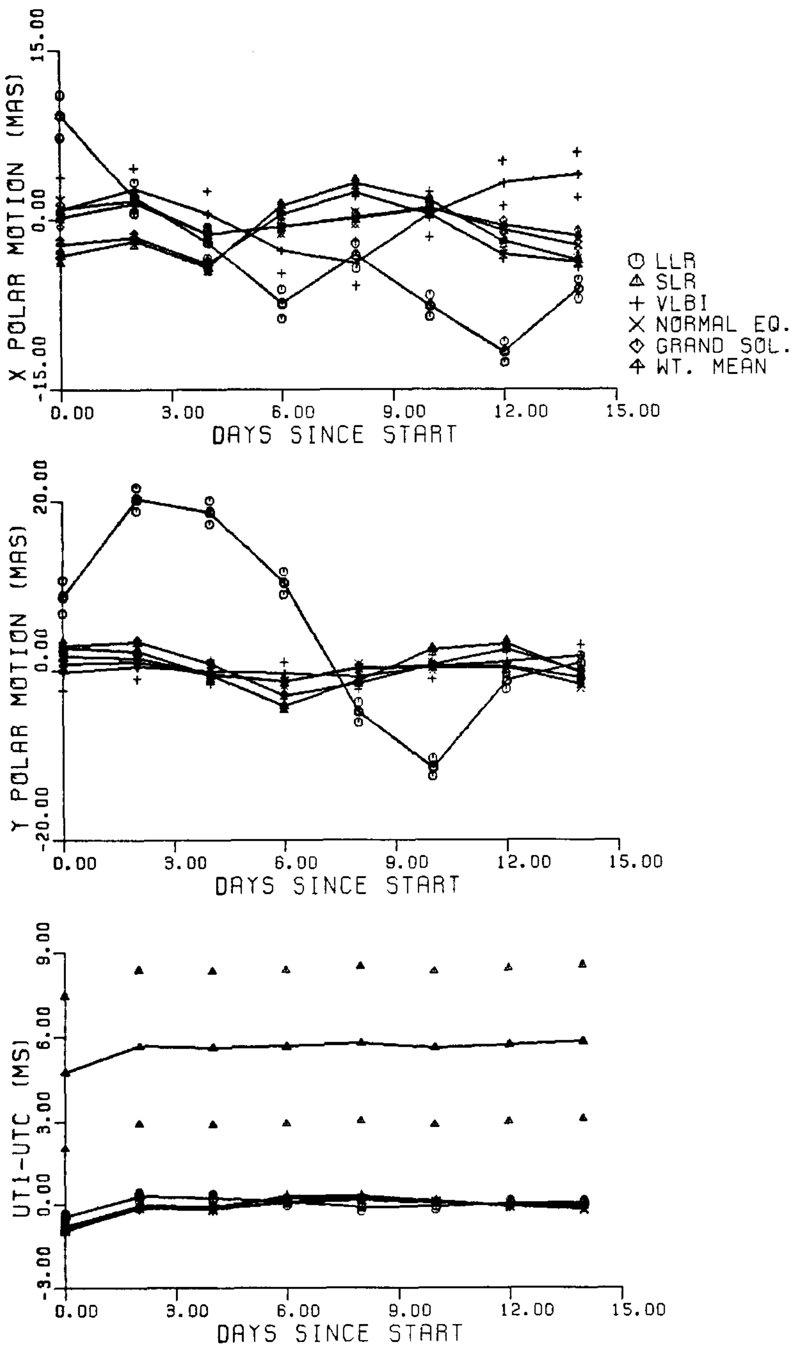

Fig. 11 Two-day ERP differences, 15 days of simulated data. Differences are from true values (simulation input). Symbols for each time series are connected by lines for clarity. Unconnected symbols above and below connected ones represent error bars. "NORMAL EQ." - Results of normal equation combination solution. "GRAND SOL." - Results of grand solution. "WT. MEAN" - Results of wt. mean solution.

Table 17 Statistical Differences, Two-Day ERP Recovery

All differences are from simulation input over 8 ERP recovery periods.

X POLAR MOTION (MAS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S. D.*
LLR	7.0(7.2)	-3.4(19.9)	-11.7(8.0)	1.2(3.4)
SLR	2.8(2.9)	-1.1(6.6)	-4.3(2.9)	0.4(1.1)
VLBI	2.7(2.7)	0.6(3.7)	3.9(2.7)	2.1(5.8)
NORMAL EQ.	1.3(1.3)	-0.2(1.1)	-2.3(1.6)	0.6(1.5)
GRAND SOL.	1.0(1.0)	-0.2(1.0)	1.5(1.0)	0.5(1.5)
WT. MEAN	2.6(2.7)	-1.4(8.4)	-4.0(2.7)	0.4(1.0)

Y POLAR MOTION (MAS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S. D.*
LLR	11.7(15.2)	5.2(132.)	20.3(15.4)	1.3(3.4)
SLR	2.4(3.2)	0.5(13.3)	-4.2(3.2)	0.4(1.1)
VLBI	0.8(1.0)	0.3(7.0)	1.7(1.3)	1.5(4.0)
NORMAL EQ.	1.1(1.4)	0.1(2.8)	1.7(1.3)	0.5(1.3)
GRAND SOL.	0.8(1.0)	0.0(1.0)	-1.3(1.0)	0.4(1.2)
WT. MEAN	2.2(2.8)	0.7(17.9)	3.4(2.5)	0.4(1.0)

UT1-UTC (MS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S. D.*
LLR	0.21(1.0)	-0.01(1.0)	-0.47(1.0)	0.12(4.8)
SLR	5.59(26.2)	5.58(494.)	5.82(12.4)	2.72(106.)
VLBI	0.38(1.8)	-0.10(8.7)	-0.97(2.1)	0.06(2.5)
NORMAL EQ.	0.35(1.6)	-0.15(13.5)	-0.92(2.0)	0.03(1.1)
GRAND SOL.	0.36(1.7)	-0.16(13.8)	-0.96(2.1)	0.03(1.0)
WT. MEAN	0.31(1.5)	-0.07(6.5)	-0.82(1.8)	0.06(2.2)

Values in parentheses show (absolute value) multiples of lowest value in column.

Note: 15 days of simulated data.

As to the average standard deviations, the results are fairly similar to that of the one-day solutions. The SLR and combination solutions continue to give the lowest values for polar motion, and the normal equation combination and grand solutions give the best for UT1-UTC. The weighted mean values for polar motion continue as the very lowest values. LLR and VLBI show about three to six times higher values for polar motion and UT1-UTC, and SLR continues with a much larger value for UT1-UTC, still reflecting the large average difference, but now only at half of that difference.

Again, considering the actual magnitudes of the recovered ERP RMS's from their correct values, we see that once again for polar motion SLR and VLBI (in X) give values of around 2.5 mas. The RMS for LLR is down only slightly to

7.0 and 11.7 mas. The combination solutions also continue at between 0.8 and 2 mas, with the VLBI Y polar motion as good at 0.8 mas. For UT1-UTC, LLR has again greatly improved, this time to 0.21 ms. VLBI and the combination solutions all have worsened slightly, to about 0.35 ms. The SLR RMS error is completely dominated by a 5.6 ms bias.

5.1.5 Five-Day ERP Results

By moving to five-day ERP recovery periods, we are beginning to use periods that are fairly long with respect to the amount of simulated data available, and as in reality, long with respect to the actual changes in real or the simulated ERP. Both for the simulated data and real data solutions, one would therefore expect the solutions to be noisier and have higher absolute RMS differences, a posteriori variances of unit weight, and standard deviations, but would expect the relative statistics to stay about the same. We also only have four such periods to examine (the first and last of which only include 2.5 days of data), greatly reducing the reliability of the statistics. However, the comparison is made anyway as the five-day period has long been in use for some SLR and most optical astrometry solutions.[6] Fig. 12 shows the usual difference curves, now quite simple due to the small number of parameters being recovered. Table 18 also shows the usual statistics, but obviously statistics with now very small degrees of freedom.

We see from the figure that SLR and both the combination solutions seem to give the best polar motion, while all of the methods except SLR are giving about the same UT1-UTC. VLBI also gives fairly good Y polar motion, while LLR still gives the poorest polar motion of all. Likewise, SLR still gives UT1-UTC with a large bias (now up to over 12 ms).

Looking at Table 18, we see that now the normal equation combination solution gives the best polar motion in both components, while that method and LLR give the best UT1-UTC. Also, as expected, the other combination solutions give the next best polar motion values, but unexpectedly, both the SLR polar motion values are slightly better than the VLBI values. The power of the VLBI solution to solve for Y polar motion seems to have changed. The LLR polar motion RMS's have again decreased relative to the other methods, but the average differences (biases) for LLR polar motion continue to increase, also giving relatively large RMS and maximum differences. For UT1-UTC, all solutions give nearly the same results, except for SLR which now has a very large average difference (bias) of 14 ms. The average standard deviations continue their trends of the shorter ERP recovery periods, except that now both the LLR polar motion and SLR UT1-UTC values are much lower than the average differences would indicate they should be.

As to the magnitudes of the RMS differences, the SLR polar motion values remained unchanged and the VLBI polar motion RMS differences have continued to increase, especially in Y, to 3.9 and 3.1 mas. And as has been the usual case, the combination solutions give results in the 1 to 2 mas range. LLR continues to give slightly decreased RMS errors in polar motion, this time

[6]The IRIS five-day solutions are not comparable to this as they are actually one-day solutions, made every five days. Likewise, the daily IRIS UT1-UTC (actually UT0) solutions are really two-hour solutions.

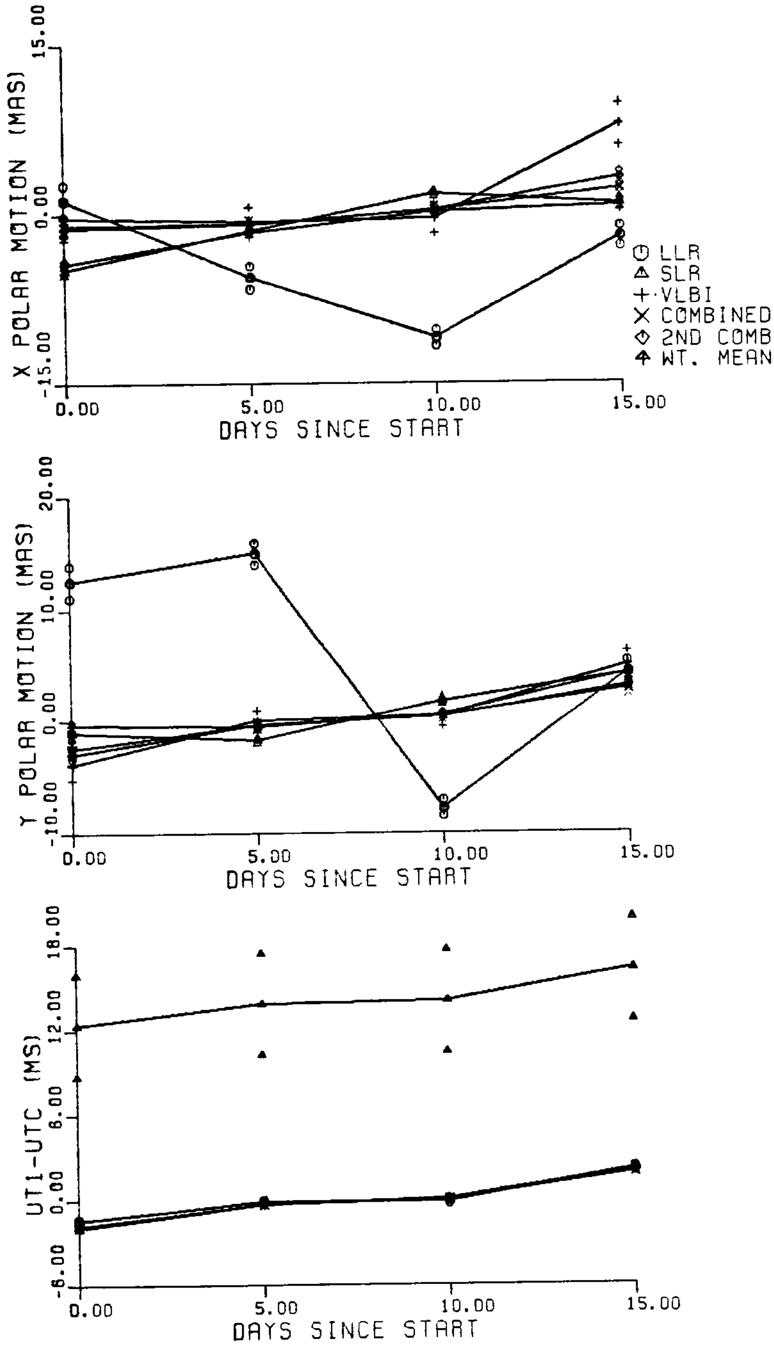

Fig. 12 Five-day ERP differences, 15 days of simulated data. Differences are from true values (simulation input). Symbols for each time series are connected by lines for clarity. Unconnected symbols above and below connected ones represent error bars. "NORMAL EQ." - Results of normal equation combination solution. "GRAND SOL." - Results of grand solution. "WT. MEAN" - Results of wt. mean solution.

Table 18 Statistical Differences, Five-Day ERP Recovery

All differences are from simulation input over four ERP recovery periods.

X POLAR MOTION (MAS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S. D.*
LLR	6.3(4.9)	-4.3(25.2)	-11.0(5.0)	1.0(3.5)
SLR	2.8(2.1)	-0.9(5.5)	-4.9(2.2)	0.3(1.1)
VLBI	3.9(3.0)	1.6(9.3)	7.8(3.6)	1.6(5.6)
NORMAL EQ.	1.3(1.0)	0.2(1.0)	2.2(1.0)	0.5(1.6)
GRAND SOL.	1.8(1.4)	0.3(2.0)	3.2(1.4)	0.4(1.5)
WT. MEAN	2.4(1.8)	-1.3(7.6)	-4.4(2.0)	0.3(1.0)

Y POLAR MOTION (MAS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S. D.*
LLR	10.7(5.8)	6.0(550.)	15.0(5.5)	1.0(3.5)
SLR	2.4(1.3)	0.8(69.0)	4.1(1.5)	0.3(1.1)
VLBI	3.1(1.7)	0.4(32.3)	4.9(1.8)	1.1(3.9)
NORMAL EQ.	1.9(1.0)	0.1(7.2)	2.7(1.0)	0.4(1.4)
GRAND SOL.	2.2(1.2)	0.0(1.0)	3.0(1.1)	0.4(1.2)
WT. MEAN	2.1(1.1)	0.9(86.0)	4.2(1.5)	0.3(1.0)

UT1-UTC (MS)

ERP Source	*RMS Diff.*	*Ave. Diff.*	*Max. Diff.*	*Ave. S. D.*
LLR	1.29(1.0)	0.11(6.4)	2.11(1.1)	0.13(6.3)
SLR	14.23(11.0)	14.16(851.)	16.31(8.7)	3.60(173.)
VLBI	1.42(1.1)	-0.04(2.6)	2.01(1.1)	0.05(2.5)
NORMAL EQ.	1.33(1.0)	-0.07(4.0)	-1.87(1.0)	0.02(1.1)
GRAND SOL.	1.37(1.1)	-0.05(3.1)	1.96(1.1)	0.02(1.0)
WT. MEAN	1.39(1.1)	-0.02(1.0)	2.03(1.1)	0.05(2.3)

Values in parentheses show (absolute value) multiples of lowest value in column.

Note: 15 days of simulated data.

to 6.3 and 10.7 mas. For UT1-UTC, all of the RMS differences have jumped substantially up to the 1.3 to 1.4 ms range, except for SLR with its large bias. The normal equation combination and grand solutions have fairly small biases in polar motion, while the other methods have biases which probably cause a substantial part of their RMS differences. For UT1-UTC, ignoring the large SLR bias, LLR has a moderate (0.1 ms) bias relative to the other methods, which are at 0.02 to 0.07 ms.

5.1.6 Comparisons of Recovered ERP Variance-Covariance Matrices

In this subsection the comparison of the overall variance-covariance matrices of the ERP solutions is considered. We have already (just above) looked at the average standard deviations for each ERP component for each ERP recovery period. However, the average and maximum standard deviations for **all of the ERP of each solution** is discussed for each of the ERP recovery periods, and in addition a summary discussion is given of the type of correlations found in the solution parameter variance-covariance matrices.

The comparison of these solutions by examining such statistics is done partly as suggested by Fedorov [1972]. The comparison of all the maximum variances, and the average standard deviations (actually the traces of the variance-covariance matrices) is for example as recommended there. The comparison of the determinants of the variance-covariance matrices, or their subtraction to check whether their results are positive definite, is not done here because: a) those values needed to compute these quantities are not easily accessible from the software used, and b) the results obtained may not be as informative as considering the actual correlations in the variance-covariance matrices (which is done instead).

Table 19 lists the trace (for completeness), the average standard deviation, and the maximum standard deviation for each ERP recovery period and method. (The average standard deviation is computed directly as the square root of the quantity: trace divided by the number of parameters.) To obtain all of these values, variances for UT1-UTC were multiplied by 225 to convert them from units of square ms to square mas so that all comparisons could be done using the same units. Relative numbers have again been given in parentheses for the standard deviations. The number of ERP recovery periods can be multiplied by three to obtain the total number of ERP parameters being considered. Orbit and radio source parameters are not included in this summary.

For all of the recovery periods, the grand solution always gives the smallest average and maximum standard deviation. The weighted mean gives the next smallest values for short (six- and 12-hour) ERP periods, while the normal equation combination solution also gives smaller such standard deviations than the individual system results alone. This is as expected since all the combination solutions contain more observations and the "square root of n" rule should approximately apply. Except for the VLBI average standard deviation for six-hour ERP, all of the individual systems give standard deviations about three or more times worse than that of the grand solution. The relative values for LLR and VLBI are comparable, except for six-hour ERP when LLR is about four to six times worse. Apparently due to the correlations of UT1-UTC with orbital parameters, SLR always gives the worst standard deviations (except for the LLR maximum standard deviation for six-hour ERP). These values are 14 to 109 times larger than that of the grand solution!

Looking at the actual values of the standard deviations shows an initial increase in the six-hour values in going to the 12-hour values, and then a gradual decrease in going up to the values for five-day ERP recovery (with SLR being an exception, as its biases apparently only increase with the ERP period length). This is explained by looking at the simulated ERP again.

Table 19 Comparison of Average and Maximum Standard Deviations

Method	Trace mas x mas	Ave. S. D. mas	Max. S. D. mas
Six-hour ERP recovery over 61 ERP recovery periods			
LLR	491.0	1.6 (9.4)	4.9 (16.7)
SLR	1043.4	2.4 (13.7)	4.1 (14.0)
VLBI	28.0	0.4 (2.2)	0.8 (2.9)
NORMAL EQ.	22.5	0.3 (2.0)	0.6 (2.1)
GRAND SOL.	5.6	0.2 (1.0)	0.3 (1.0)
WT. MEAN	10.1	0.2 (1.3)	0.4 (1.4)
12-hour ERP recovery over 31 ERP recovery periods			
LLR	431.7	2.1 (3.1)	5.5 (4.5)
SLR	10087.9	10.3 (15.2)	18.0 (14.5)
VLBI	418.2	2.1 (3.1)	4.3 (3.5)
NORMAL EQ.	59.0	0.8 (1.2)	1.5 (1.2)
GRAND SOL.	43.7	0.7 (1.0)	1.2 (1.0)
WT. MEAN	55.8	0.8 (1.1)	1.5 (1.2)
One-day ERP recovery over 16 ERP recovery periods			
LLR	174.6	1.9 (2.9)	3.0 (2.7)
SLR	15848.3	18.1 (27.5)	31.5 (28.9)
VLBI	252.9	2.3 (3.5)	4.3 (3.9)
NORMAL EQ.	23.8	0.7 (1.1)	1.2 (1.1)
GRAND SOL.	20.9	0.6 (1.0)	1.1 (1.0)
WT. MEAN	30.0	0.8 (1.2)	1.5 (1.4)
Two-day ERP recovery over eight ERP recovery periods			
LLR	53.9	1.5 (3.3)	2.0 (2.8)
SLR	13367.5	23.6 (51.2)	40.9 (56.3)
VLBI	61.6	1.6 (3.5)	2.9 (4.0)
NORMAL EQ.	5.8	0.5 (1.1)	0.8 (1.1)
GRAND SOL.	5.1	0.5 (1.0)	0.7 (1.0)
WT. MEAN	8.1	0.6 (1.3)	1.1 (1.5)
Five-day ERP recovery over four ERP recovery periods			
LLR	23.9	1.4 (3.8)	2.0 (4.1)
SLR	11656.7	31.2 (84.1)	54.0 (109.)
VLBI	18.5	1.3 (3.4)	1.9 (3.8)
NORMAL EQ.	2.1	0.4 (1.1)	0.6 (1.1)
GRAND SOL.	1.6	0.3 (1.0)	0.5 (1.0)
WT. MEAN	2.7	0.5 (1.3)	0.8 (1.6)

Notes:
1. Values in parentheses show (absolute value) multiples of lowest value in column.
2. Variances for UT1-UTC were converted from square ms to square mas before computing all values.
3. Summary for solutions with 15 days of simulated data.

For six-hour recovery, the ERP can be recovered with the same fluctuations with which it was simulated. For longer periods, the recovered values are actually averages of changes which still occur in the data every six hours. In effect, we have introduced a model error by not always recovering the ERP over the same periods at which it exists in the data (six hours). As the recovered ERP period increases greatly from six hours, the six-hour fluctuations average out more, giving smaller standard deviations again (but never as small as at six hours). **This strongly emphasizes the importance of using ERP recovery periods consistent with the periods of change in the actual ERP.** Otherwise a modeling error (for the ERP recovery) is being committed.

Finally we look at the ranges or at least maximum sizes of the correlations between parameters. Table 20 gives such a summary of those correlations which are significant. This table shows the maximum or range of (the absolute values of) all correlations greater than 0.2. We have divided the correlations first according to solution method and then parameter type. Correlations with the lunar and Lageos orbit parameters are included. Correlations with and among radio source positions were all less than 0.2. Due to software limitations, these orbit and source position parameter correlations were not available in the combination solutions. Correlations were ignored in the weighted mean solutions (as is commonly done in practice).

Immediately obvious is the greater number of significant correlations for the individual systems than for the combination solutions. The only significant correlations in the combination solutions were among the polar motion and UT1-UTC parameters during the same period. These were nearly the same for both the normal equation combination and grand solutions, with values of 0.2 to 0.6. Even correlation among polar motion and UT1-UTC at different times was negligible. VLBI gave similar results, except with generally higher correlations (0.5 to 0.8), and with five-day ERP recovery, correlations of polar motion with UT1-UTC at other times, of up to 0.3. The lunar and Lageos orbit parameters showed wide-ranging correlation among themselves, ranging from 0 to 1. Unlike any other method, SLR showed negligible correlation among polar motion parameters. However, orbit parameter correlations with polar motion were noticeable for six- and 12-hour ERP (0.3 to 0.6) and with UT1-UTC for 5 day ERP (0.2-0.3). The correlations of UT1-UTC with UT1-UTC of other periods, and with X and Y orbit components was always quite high however, from 0.975 to 1 in all cases. This again clearly demonstrates the poor separability of UT1-UTC parameters from orbit (XY plane) orientation parameters, but shows that the correlation decreases slightly from 1 as the ERP period becomes shorter. The LLR solutions have a wide range of significant correlations, but no extremely high ones except among the orbit parameters, and between polar motion and UT1-UTC if six-hour ERP recovery is done. The correlations of UT1-UTC with UT1-UTC of other periods, and with Z axis orbit parameters increases with ERP period, from near 0.5 or 0.6 to 0.9. The correlations among polar motion parameters are, however, similar or slightly less than the VLBI values.

Table 20 Summary of Range of Correlations Between Parameters

	Parameters	6 hours	12 hours	1 day	2 days	5 days
LLR						
	X vs. Y	.76	.1-.7	.6	.6	.6
	XY vs. UT1	.97	.2-.5	.4	.4	-
	XY vs. other UT1	.3	-	.4	-	-
	UT1 vs. " UT1	.5	.5	.7	.82	.82-.91
	XY vs. SVXY	.4	-	-	-	-
	XY vs. SVZ	.67	.48	-	-	.3
	UT1 vs. SVXY	.73	.3-.8	.85	.8-.9	.88-.96
	UT1 vs. SVZ	.95	.1-.999	.998	.998	.998
SLR						
	UT1 vs. o UT1	.988-.996	.998-.999	.999-1	1	1
	XY vs. SVXY	-	-	-	-	-
	XY vs. SVZ	.57	.3	-	-	-
	UT1 vs. SVXY	.975-.998	.995-.999	.999-1	.999-1	1
	UT1 vs. SVZ	-	-	-	-	.2-.3
	SV	.999	.1-.9	.1-1	.1-1	.2-1
VLBI						
	X vs. Y	.5-.81	.5-.8	.7-.8	.73-.75	.73-.75
	XY vs. UT1	.5-.78	-	-	.5-.6	.52-.56
	XY vs. other UT1	-	-	-	-	.3
Normal Equation Combination						
	X vs. Y	.4	.6	.4-.6	.3-.5	.4-.5
	XY vs. UT1	.6	.5	-	-	-
Grand Solution						
	X vs. Y	.2-.5	.34-.55	.35-.42	.39-.44	.39-.44
	XY vs. UT1	.2-.6	.4-.5	.36-.50	-	-

Notes:

1. Maximum or range of absolute value of correlations shown.
2. Correlations below 0.2 not shown (not listed, or "-" given).
3. Abbreviations: "X", "Y" - polar motion, "UT1" - UT1-UTC, "SV" Cartesian state vector for Moon (LLR) or Lageos (SLR). ("SVXY" implies X-Y plane SV parameters. "SVZ" implies Z axis SV parameters.)
4. Correlations between ERP and state vectors/radio source positions not available in combination solutions due to software limitations.

5.2 Comparisons of Data Amounts, Computation Time

Although looking at their accuracy and precision may be the primary way in which to decide which of several methods of ERP determination is best, it is also of interest to compare the actual amount of data being handled in each of these methods and the computation times involved. As to rigorous estimates of these values, we are limited by having done a simulation, and therefore cannot account well for data preprocessing, the more complete modeling used to process real data, and the varying amounts of data delivered by an observational system. However, by looking at the simulations we can point out some items which may be of interest, especially as to the relative differences between the various methods. A summary of these points will also be presented in the next chapter.

5.2.1 Comparison of Amounts of Input Data

First, we look at the amounts of "data" that have gone into each of our solutions, whether it be the simulated observations, or the values generated from them, the normal equations, or the ERP. These amounts are shown in Table 21, and represent all the "data" used in any of our ERP solutions over the 15-day period of the simulated data.

For LLR, SLR, and VLBI, we have given for each ERP period recovered:

1. the number of observations and number of (eight-bit) bytes they take up in the GEODYN binary format [Eddy et al., 1983, Appendix C.7],

2. the number of bytes used to contain the normal equations of each solution in the GEODYN E-matrix format [Eddy et al., 1983, Appendix C.17], and

3. the number of ERP parameters, with an estimated number of bytes needed to store them, as resulting from each system's solution.

The limitations of these results include:

1. Only the LLR and SLR normal points are listed. The actual data amounts may be several hundred times these already very large values.

2. The VLBI observations of delay and delay rate only are being considered here. In practice, the true raw VLBI data consists of many gigabytes of data which must be correlated to obtain the delays and delay rates.

3. The GEODYN binary format is by its nature, very general and certainly not the most common or efficient for storing such data.

4. The GEODYN E-matrix format also possibly could be optimized better.

5. With real data processing, many other parameters would be included in the solutions, thereby substantially increasing the size of the normal equations.

Table 21 Comparison of Amounts of Input Data for 15-Day Solutions

	System	Data (obs., bytes)	Normals (bytes)	ERP (parms.,	bytes)
LLR					
	6 hours	2030, 146160	155724	183	2196
	12 hours	same	46284	93	1116
	1 day	same	15864	48	576
	2 days	same	6264	24	288
	5 days	same	3192	12	144
SLR					
	6 hours	24489, 1.68 mb	155724	183	2196
	12 hours	same	46284	93	1116
	1 day	same	15864	48	576
	2 days	same	6264	24	288
	5 days	same	3192	12	144
VLBI					
	6 hours	14086, 990 kb	202764	183	2196
	12 hours	same	73164	93	1116
	1 day	same	32664	48	576
	2 days	same	17688	24	288
	5 days	same	11928	12	144

Notes:
1. "bytes" for the data is computed as the number of observations times 72 bytes/observation (as in the GEODYN binary format).
2. "bytes" for the normal equations is the number of bytes used to store the normals in GEODYN E-matrix format.
3. "parms." for ERP is the number of ERP recovery periods times three (for X and Y polar motion, and UT1-UTC).
4. "bytes" for ERP is determined from the number of parameters times three (for time, parameter values, and standard deviation) times four bytes.
5. Additional information, such as station reports, problem reports, calibration data, model information, etc. is not considered. Delay rates are included in the VLBI observations.

Keeping these limitations in mind, we make the following points:

First, we can say little about which system would have the least amount of data in practice, as we have assumed the highest possible data rates in the simulations. In reality, the effect of weather and equipment malfunctions is too unpredictable. However, we can possibly generalize that under good operating conditions LLR will have the least data, while SLR will have the most. Also it is clear that the task of transmitting the data of any of these systems to another user would not be a trivial task, but would require at least a moderately large amount of data transmission resources or be done by magnetic tape under ordinary conditions.

As to the normal equations, we see that of course their size depends heavily on the number of parameters being solved for. Clearly, when the parameter set is large (or if further model parameters were added as they would be with real data), the size of the normals may be quite substantial. With only station coordinates, a state vector, and six-hour ERP as parameters, the LLR normals are already larger than the data set! For SLR and VLBI the normals are also substantial in size in comparison to the data (9% and 20% respectively). Only when the number of ERP are dropped to daily or longer periods does the size of the normals become fairly small compared to the data.

Finally, we see that the number of the ERP (and their storage space) generated by each solution is always fairly small compared to the size of the normals, and nearly negligible when compared to the amount of data.

5.2.2 Comparison of Amounts of Computer Time

Secondly, we can examine the amounts of computer time used in the simulation solutions to see which methods may be the least computer intensive. The CPU times for all of the 15-day solutions are given in Table 22.

For each ERP method we show the CPU time in seconds on the Ohio State IRCC IBM 3081D computer. Times are shown both to do solutions and to set up the normal equations (where applicable) for each ERP period. Since GEODYN currently does not allow **both** solutions and normal equation setups, the "solution" and "normals only" times do indeed correspond to two different computer runs.[7] The weighted mean solutions are done in the PLOTERP program (previously discussed in section 4.2.3), but the times given here are very pessimistic as they include the program compilation and loading, and setup of plots. One to three seconds would be more realistic values. The "weighted mean with solutions" values are the total time for the LLR, SLR, VLBI, and weighted mean solutions. Likewise, the "normal equation combination with normals" times are the total times for the LLR, SLR, and VLBI normal equation setups in GEODYN and their solution in SOLVE. The time for the grand solution includes that of the "normal equation combination solution with normals" plus that of setting up the normals again and getting a new solution (in SOLVE).

Once again, we have some limitations on the conclusions we can draw from this table:

1. These times, especially the larger ones, are fairly approximate. This is because the CPU time required depends on the current overall load on the computer, since programs are switched in and out of the execution (requiring extra CPU time) when loads are heavy. This is why some of the solutions which would be expected to require less time than another end up requiring slightly more.

2. Once again, the compression of the laser raw data into normal points is not being considered, and correlator processing required to obtain VLBI delays and delay rates from the raw observation tapes is of course not included.

[7] For practical use, a minor software change should be made to allow both to be done at the same time.

Table 22 Comparison of Computer Time for 15-Day Solutions

System	ERP Recovery Period 6 hours	12 hours	1 day	2 days	5 days
LLR					
solution	284	260	254	256	254
normals only	192	193	190	191	190
SLR					
solution	1210	1188	1164	1177	1166
normals only	887	880	870	876	878
VLBI					
solution	266	175	168	160	159
normals only	76	68	65	64	65
Weighted Mean					
solution	7	7	5	4	4
wi. sol.	1767	1630	1591	1597	1583
Normal Equation Combination					
solution	15	3	2	1	1
wi. normals	1170	1144	1127	1132	1134
Grand Solution					
solution	1969	2260	2240	2244	2247

Notes:
1. All times are CPU seconds on the IRCC IBM 3081D.
2. GEODYN 8210.7 used for LLR, SLR, VLBI solutions and setup of normals, SOLVE 8212.0 used for the data combination solution, and PLOTERP for wt. mean solution.
3. The LLR, SLR, and VLBI solutions were done with three (outer) iterations. The normal equation combination solution is a one-iteration solution, while the grand solution has, in effect, two (outer) iterations.
4. The VLBI values were doubled to account for delay rate observation processing.
5. The "wi. sol." and "wi. normals" include the times for the LLR, SLR, and VLBI solutions and normal equation setups respectively.

3. Models used to process real data would be greatly expanded, thus greatly increasing the processing time. This is especially true of the SLR and perhaps the LLR solutions, where the orbital modeling would normally be quite complex.

4. Certainly more efficient programs or versions of these programs, or even more efficient computers, could be used for the data processing. For example, GEODYN II, which can take advantage of vector processing on a CDC Cyber computer would be substantially faster than the GEODYN version used here. In any case, the relative processing times would likely still be similar.

5. The number of outer iterations vary, with the LLR, SLR, and VLBI solutions each having three, the normal equation combination solution (by definition) having one, and the grand solution having in effect two.

With these limitations in mind, the following points can be made:

1. The differences in CPU time between solutions of different ERP recovery time periods is fairly small, at least in comparison to the total amount of time for these solutions.

2. The VLBI solutions take substantially less time than the SLR and to a limited extent the LLR solutions, due to the orbital computations of the laser ranging solutions. This of course does not take into consideration that the VLBI correlation process is far more computer intensive than the laser ranging normal point computations.

3. The complete normal equation combination solution (including creating the normals) takes considerably less time than the complete weighted mean solution (including the individual system solutions). However, this is with the disadvantage that the normal equation combination solution will probably not be a truly converged solution and its results may be worse than the weighted mean or other solutions. See, for example, the comparisons of the a posteriori variance of unit weight in Appendix A.

4. Both the solutions by normal equations, and especially by the weighted mean take negligible computer time. Even when the individual system solutions and normal equations setups are added in, there is still little difference between these methods.

6. CONCLUSIONS

In this final chapter, a summary is given of the results of this study, some comments are made concerning the advantages of normal equation combination and grand solutions, overall conclusions are given, and finally, suggestions for further work are presented.

6.1 Summary of Results

Before providing a summary of the important results of Chapter 5, we should review some of the assumptions made in doing the simulation experiments, in order to consider the possible limitations of these results. First, we recall that only the overall geometry of these observations were considered. Second-order, or "systematic" effects (modelable and unknown) have been ignored, with only random noise added to the simulated observations. Second, we have assumed that data is available at the highest possible rates, rates that will likely never be achieved in practice, but assuming good weather, could be closely approached for short periods. Third, we have assumed the use of two SLR/LLR stations (Simeiz and Richmond) that may not contribute any observations on a regular basis (with Simeiz not yet operating as an LLR station, and with the possibility that the Richmond station will never be built).

The probable effects of these assumptions are:

(1) It is expected that the relative accuracies of the various methods will not be greatly affected by such problems. But the accuracy estimates themselves will clearly be overoptimistic since "systematic" effects have been ignored, especially errors currently unmodeled in practice. It is also possible that some of the other second-order effects are highly correlated with the ERP or other parameters, which could slightly increase the amount of error in those parameters. It is also true that as the various models for each system regarding these "systematic errors" are improved, the accuracies estimated here in the simulations will become more realistic.

(2) The fact that we have assumed extremely high data rates will have one main effect, that solutions for especially short (one day or less) ERP periods will not generally be as good (or perhaps not even possible) for the laser systems, and VLBI solutions will only be possible during the days (now every five days) of continuous IRIS observations. However, the combination methods of course can be used to obtain solutions when data of any type exists. In any case we have clearly demonstrated the maximum possible accuracies obtainable with the currently operating systems (without substantially adding to the number of stations or increasing the observational precision).

(3) The lack of the two SLR/LLR stations in the currently operating stations is probably not particularly significant as regards SLR. These stations apparently do not provide significantly better station geometries in Europe or North America (and besides, many other operating SLR stations on those continents were ignored in this study). Additionally, Schutz et al. [1985] have shown that as few as eight SLR stations may be sufficient to determine ERP values. However, the lack of these two stations for LLR should significantly degrade the quality of the LLR network. It really should be shown by further simulation that the remaining four LLR stations are capable of providing the excellent UT1-UTC values obtained in some of the simulations. The simulations of Larden [1982] do however tend to indicate that this is true. Perhaps the greater loss in not having these two combination SLR/LLR stations is that it would greatly reduce the number of available colocations between the various systems.

6.1.1 Accuracy and Precision Obtained for Various ERP Periods

With the above limitations in mind, we can make a brief summary of the different statistical results. Tables 23, 24, 25 and 26 summarize respectively the relative RMS differences, average differences, maximum differences, average standard deviations of the ERP recovered, by all ERP recovery periods, methods, and ERP parameter type. Table 27 summarizes the average and maximum standard deviations for all ERP recovery periods and methods. In these tables the best methods are designated with a "*", and methods with similar results or only slightly worse results are shown with a "+" or "-" respectively.

The summary of relative RMS differences in Table 23 shows mainly that the combination solutions are clearly capable of providing the best accuracies. The grand or normal equation combination solutions each appear capable of providing the smallest RMS differences in most cases, with the weighted mean and even VLBI providing values at most two to three times worse. LLR also provides the best UT1-UTC for one, two, and five days' values, but little else as well as the other methods. VLBI provides some of the best Y polar motion (due to its station geometry) and SLR fairly consistent (although never the best) X polar motion.

As to the average differences (biases) occurring in the ERP results, Table 24 (and the tables of the previous chapter) shows that as expected SLR has substantial biases in UT1-UTC, but only in the one-day to five-day ERP period recovery. SLR actually gives the most bias free UT1-UTC for six-hour ERP recovery. LLR also always has the worst polar motion mostly due to its large biases relative to the other methods. For the two- and five-day solutions, SLR, VLBI, and the weighted mean solutions also begin to give noticeable biases in polar motion. The normal equation combination and grand solutions, although giving some noticeable biases in UT1-UTC, seem generally to have biases smaller or similar in size than the other methods. LLR appears to be capable of giving the most bias free UT1-UTC, with the weighted mean solutions providing similar results. It appears that the large UT1-UTC bias in SLR clearly affects the normal equation combination and grand solutions, but not the weighted mean solutions.

Table 23 Relative RMS Differences for All Methods and ERP Recovery Periods

Recovery Period	LLR X	LLR Y	LLR U	SLR X	SLR Y	SLR U	VLBI X	VLBI Y	VLBI U	Weighted Mean X	Weighted Mean Y	Weighted Mean U	Normal Eq. X	Normal Eq. Y	Normal Eq. U	Grand Sol. X	Grand Sol. Y	Grand Sol. U
6 hours				+	+		-	-	-	+	+	-	*	*	*	*	*	+
12 hours			-	-	+	+	+	*	+	+	+	+	*	+	*	*	+	*
1 day			*	-			-	+	+	-	-	*	+	+	*	*	*	*
2 days			*	-			-	*	+	+	+	+	+	+	+	*	*	+
5 days			*	-	+		-	+	+	+	+	+	*	*	*	+	+	+
all				-			-	-	-	-	-	-	+	+	+	+	+	+

Notes:

X - X polar motion; Y - Y polar motion; U - UT1-UTC "normal eq." is the normal equation combination solution.

* best method(s) (smallest RMS difference)
\+ RMS difference multiple is between 1 and 2.
\- RMS difference multiple is between 2 and 3.
(blank) RMS difference multiple is greater than 3.

Table 24 Relative Average Differences for All Methods and ERP Recovery Periods

Recovery Period	LLR X	LLR Y	LLR U	SLR X	SLR Y	SLR U	VLBI X	VLBI Y	VLBI U	Weighted Mean X	Weighted Mean Y	Weighted Mean U	Normal Eq. X	Normal Eq. Y	Normal Eq. U	Grand Sol. X	Grand Sol. Y	Grand Sol. U
6 hours						*			-	*		-		+	+		*	+
12 hours			*	-	*		-		*	-	*	+	+		+	*	-	+
1 day			*					-				+	*	+			*	
2 days			*										+	-		*	*	
5 days									-			*	*			+	*	
all																	-	

Notes:

X - X polar motion; Y - Y polar motion; U - UT1-UTC "normal eq." is the normal equation combination solution.

* best method(s) (smallest average difference)
\+ average difference multiple is between 1 and 2.
\- average difference multiple is between 2 and 3.
(blank) average difference multiple is greater than 3.

The relative maximum differences summarized in Table 25 merely tend to confirm the RMS difference results. The grand and normal equation combination solutions are clearly the best, with the weighted mean and VLBI solutions usually having values one to three times higher. The strength of LLR for UT1-UTC, the consistent SLR X polar motion, and VLBI poor X and good Y polar motion continue to be obvious.

Table 25 Relative Maximum Differences for All Methods and ERP Recovery Periods

Recovery Period	LLR			SLR			VLBI			Weighted Mean			Normal Eq.			Grand Sol.		
	X	Y	U	X	Y	U	X	Y	U	X	Y	U	X	Y	U	X	Y	U
6 hours				+	+			+	–	+	*	–	+	+	*	*	+	+
12 hours			–	–	+	+	+	+	+	–	+	+	+	+	+	*	*	*
1 day			*	–			+	+	–	+		+	+	+	+	*	*	+
2 days			*	–			–	+	–	–	–	+	+	+	+	*	*	–
5 days			+	–	+			+	+	+	+	+	*	*	*	+	+	+
all				–				+	–	–	–	–	+	+	+	+	+	–

Notes:

X - X polar motion; Y - Y polar motion; U - UT1-UTC "normal eq." is the normal equation combination solution.

* best method(s) (smallest maximum difference)
+ maximum difference multiple is between 1 and 2.
- maximum difference multiple is between 2 and 3.
(blank) maximum difference multiple is greater than 3.

The relative average standard deviation summary shown in Table 26 indicates that the weighted mean solution usually give the lowest polar motion standard deviations, while the grand solution always gives the lowest UT1-UTC standard deviations. The weighted mean standard deviations probably appear so optimistic because no correlations are considered for that solution. The normal equation combination ERP and the SLR polar motion standard deviations are all usually within a factor of two of the lowest values. VLBI and the weighted mean solutions provide values within a factor to three. All LLR ERP and the SLR UT1-UTC standard deviations are quite large in comparison, due to the large biases which can (and do) exist for parameters determined in those solutions.

Instead of looking at the average standard deviations by parameter type, Table 27 summarizes the average and maximum standard deviation for all ERP parameters, regardless of type. Clearly, and as expected, the grand solution always provides the smallest values. The weighted mean and normal equation combination solutions provide values normally only one to two times as high. The individual systems only sporadically were capable of values even as little as three times as high.

Table 26 **Relative Average Standard Deviation for All Methods and ERP Recovery Periods**

Recovery Period	LLR X	LLR Y	LLR U	SLR X	SLR Y	SLR U	VLBI X	VLBI Y	VLBI U	Weighted Mean X	Weighted Mean Y	Weighted Mean U	Normal Eq. X	Normal Eq. Y	Normal Eq. U	Grand Sol. X	Grand Sol. Y	Grand Sol. U
6 hours				+	+		–	–	+	+	+	+	+	+	+	*	*	*
12 hours				+	+				–	*	*	+	+	+	+	+	+	*
1 day				+	+				–	*	*	–	+	+	+	+	+	*
2 days				+	+				–	+	*	–	+	+	+	*	+	*
5 days				+	+				–	*	*	–	+	+	+	+	+	*
all				+	+				–	+	+	–	+	+	+	+	+	*

Notes:

X - X polar motion; Y - Y polar motion; U - UT1-UTC "normal eq." is the normal equation combination solution.

* best method(s) (smallest average std. dev.)
+ average std. dev. multiple is between 1 and 2.
- average std. dev. multiple is between 2 and 3.
(blank) average std. dev. multiple is greater than 3.

Table 27 **Relative Standard Deviation for All Methods and ERP Recovery Periods**

Recovery Period	LLR A	LLR M	SLR A	SLR M	VLBI A	VLBI M	Weighted Mean A	Weighted Mean M	Normal Eq. A	Normal Eq. M	Grand Sol. A	Grand Sol. M
6 hours					–	–	+	+	+	–	*	*
12 hours							+	+	+	+	*	*
1 day	–	–					+	+	+	+	*	*
2 days		–					+	+	+	+	*	*
5 days							+	+	+	+	*	*
all							+	+	+	–	*	*

Notes:

A - average ERP standard deviation
M - maximum ERP standard deviation
"normal eq." is the normal equation combination solution.

* best method(s) (smallest standard deviation)
+ standard deviation multiple is between 1 and 2.
- standard deviation multiple is between 2 and 3.
(blank) standard deviation multiple is greater than 3.

Keeping in mind that they do not represent realistic values, but the best possible values (without systematic errors in the observations), Table 28 also shows the best absolute accuracies obtained. This table shows primarily (e.g., in the 12-hour to five-day solutions) that accuracies are possible of approximately 1 mas for polar motion, and 0.2 ms for UT1-UTC. This compares well with some of the currently estimated accuracies for ERP determination, as shown in Table 1. For example, the best polar motion values are currently accurate at the 2 mas level, with the simulation experiments' results showing that 50% better accuracy may be possible. Likewise, the current accuracies of 0.1 to about 1.0 ms for several methods of UT1-UTC determination matches the simulations' 0.2 ms values well. However, special note should be made of the accuracies listed for the six-hour ERP period, at about 0.2 mas for polar motion, and 0.01 ms for UT1-UTC. Why are these values nearly an order of magnitude better than the others, either in the simulations or real results? The answer would again appear to be connected with how the data were simulated. For the six-hour ERP, each six hours of data had a constant ERP set used to simulate them, which apparently has been recovered with great accuracy. For the other ERP periods, the reference "actual" ERP values were indeed constant over their periods, but since the same simulated data was in use, the data reflects (slight) changes in the ERP every six hours. As noted in 5.1.6, if we try to recover a constant ERP value that in fact is changing during the period we assume it constant, the RMS will naturally be higher.

Table 28 Solution Methods Providing Least RMS Difference ERP Values

Recovery Period	X polar Motion (mas)	Y polar Motion (mas)	UT1-UTC (ms)
6 hours	n. eq./grand 0.2	n. eq./grand 0.2	n. eq. 0.01
12 hours	n. eq./grand 1.7	VLBI 1.4	n. eq./grand 0.20
1 day	grand 1.4	grand 0.9	see note 2 0.26
2 days	grand 1.0	VLBI/grand 0.8	LLR 0.21
5 days	n. eq. 1.3	n. eq. 1.9	LLR/n. eq. 1.3
average	1.1	1.0	0.40

Notes:
1. "n. eq." represents the normal equation combination solution and "grand" represents the grand solution.
2. For one-day UT1-UTC recovery, the LLR, weighted mean, normal equation combination, and grand solutions all had the same best value.

In the case of the six-hour ERP recovery, the ERP did not change during the period, and therefore a very small RMS results. This means that if the real polar motion or UT1-UTC values are constant over the ERP period being covered, then it is possible that such high accuracies could be obtained. In reality, probably mostly due to atmospheric influences, actual short-period ERP measurements have shown that the ERP do change rapidly over such short periods (although polar motion apparently fluctuates little in comparison to UT1-UTC, i.e., it has less spectral power at high frequencies) [Robertson et al., 1985; Robertson and Carter, 1985, pp. 302-305; Carter and Robertson, 1985b].

A few other miscellaneous conclusions can also be drawn from the simulations:

(1) As long as fairly reasonable approximate values (within several multiples of the weights) are used, the results of most of the solutions seem unaffected. Exceptions are the six-hour ERP recovery by LLR, and over all periods by the normal equation combination solution. The problem with the latter solution is that it is not a fully converged solution, so although it may provide relatively accurate values for parameters, they are not the best possible, and relatively large a posteriori variances of unit weight and (hence) standard deviations also result.

(2) Six-hour ERP values, and probably three-hour ERP values can be recovered with high accuracy from SLR and VLBI observations (assuming enough data is present). LLR on the other hand has noticeably worse ERP, when recovery is made over six- and possibly 12-hour periods, and substantially higher correlations between parameters than the other methods.

(3) SLR may be capable of providing useable UT1-UTC for six- and 12-hour recovery periods. This is demonstrated by the small average and other differences and standard deviations, and slightly lower correlations with the orbit parameters when recovering ERP of those periods.

6.1.2 Results Concerning Data Amounts and Computation Times

A summary of our other results from Chapter 5 which do not deal with the accuracy or precision of the various methods is presented here. To understand the importance of these results, we consider the point of view of an Earth rotation service, such as the BIH, NEOS, or the future IERS. The interest of any Earth rotation service is not only to provide ERP of the highest possible accuracy but also to minimize the amount of data transmission and computation time in order to do so. This minimization may not be too important for ERP determination which is done long after the observations have been made (e.g., by transferring data on magnetic tape by normal mail, and using "off hours" computer time), but they become of possibly great importance for so-called "quick look" results which are needed as quickly as possible.

Concerning the data amounts, we can look at the three possible types of "data" and summarize the results as follows:

(1) Ignoring data preprocessing (normal point computations for LLR and SLR, correlator processing for VLBI), the SLR network is capable of

generating the largest amount of data, with VLBI generating only just over half as much (delay and delay rate data), and LLR about a tenth as much. This is still under the assumption of the highest possible data rates. In any case, it is likely that anywhere from several hundred to several thousand records (i.e., nearly card images) would be generated by each system, even with much less observing. Since some of this would be sent daily, instead of being accumulated over the 15 days of data here, this amount could feasibly be sent via an electronic mail system, although at the normally available 1200 baud transmission rates this could be expensive and perhaps very time consuming.

(2) The size of the normal equations depends little on the observational system but is almost entirely dependent on the number of ERP (and other parameters) being solved for. For long ERP recovery periods (two or five days) the size is fairly small, but as the ERP period shortens to six hours (or as additional parameters are added such as would be in practice) the size will increase greatly, easily exceeding that of the original data, which is already at its highest possible levels. Unless the number of parameters is kept small, the transmission of the data itself would probably be just as economical.

(3) The amount of data in an ERP series itself is always fairly trivial compared to the amount of data or normal equations used to generate them. As is commonly done now in practice, the transmission of this data by electronic mail would be a very low cost procedure.

Examining the computer time results allows us to draw several conclusions:

1. The actual weighted mean solutions, or combination and solution of normal equations, require small amounts of computer time. It is the creation of the normal equations and the individual system solutions which require large amounts of time.

2. If individual solutions are to be done anyway, the saving of the normal equations if possible later use were likely would result in a great savings of computer time.

3. The normal equation combination solution (being a "single iteration" solution) requires much less time than the fully iterated individual solutions and than their weighted mean combination solution. As noted already, this is at the expense of a not completely converged solution.

4. The ERP recovery period affects the total computer time very little.

5. Ignoring data preprocessing (again), it is clear that SLR and then the LLR individual solutions or normal equation setups are very computer intensive, with the VLBI solutions less so. With real data solutions, all of these computations would take even longer due to the additional modeling which would be done, especially for the LLR and SLR computations which would have much more extensive orbital models.

6. Considering that the primary program in use (GEODYN) is fairly efficient and the speed of the computer used (an IBM 3081D), the computer time for the individual solutions and/or normal equation setups are quite large (e.g., 20 minutes for the SLR solution with simple models). Mini- or microcomputers would not efficiently be able to do the individual solutions and/or normal equation setups, but only the normal equations combination or some type of weighted mean solution of ERP series obtained from elsewhere. And, if the normal equations of all these systems are set up at one time, even a large mainframe might be pressed to accomplish such a task, unless program efficiency was increased or a vector or array processor was in use. In practice, doing solutions every few days instead of with 15 days of data may reduce this problem somewhat, and the use of vector or array processing would substantially eliminate it.

6.2 Advantages of Normal Equation Combination and Grand Solutions

After studying at length the idea of combination solutions, particularly by the combination of normal equations, several advantages of this method or the use of grand solutions over other methods of ERP determination have become obvious which cannot be very well quantified. Instead a simple list of these is given.

1. In order to combine the normal equations, the models and approximate values used must be carefully matched. This makes it necessary to make the models for each observational system consistent with each other, and assures that recovered parameters are indeed truly compatible with each other (e.g., all in one unified reference system at one scale, with the same constants in use, etc.). Also if the individual systems' normals are then solved, the results can be compared knowing the same models, constants, etc. are in use. Such comparisons are only currently possible if each systems' software uses the same set of standards (i.e., the MERIT Standards [Melbourne et al, 1983]).

2. Combining normal equations even allows us to combine equations that could not be solved on their own, i.e., singular sets of equations can sometimes be added and a solvable set obtained. If applied carefully, i.e., if the user checks that the final system is really nonsingular, this might be a useful feature. For example, when one or more of the systems has a small amount of data, be it a single satellite pass at one station, single station LLR data, or one baseline VLBI data, the data can still be combined together so that if enough is available overall a solution can be obtained. This technique is extremely powerful in that it may allow the handling of periods of sparse data from any or all systems, possibly even when no solution can be made from each of the systems involved alone.

3. By combining data from different systems, we end up obtaining better values for parameters that may normally be highly correlated with other parameters. For example, it has been shown here that SLR normally cannot give UT1-UTC without biases (at least with one- to five-day recovery periods) due to the inseparability (high correlation) of UT1-UTC with the orientation of Lageos' orbit. However, if we do a combination solution, the UT1-UTC value is forced to its correct solution by the LLR and VLBI data and the Lageos orbit parameters are also then improved as well.

Strengthening the orbit could in turn strengthen other model parameters (if they are included) such as gravity field coefficients, station coordinates, etc. For "quick look" solutions, it is not likely that many such parameters would be solved for, but for "final" long arc solutions, the additional accuracy obtainable for many parameters might be very important. For the ERP themselves, the strengths of each individual ERP method (e.g., LLR for UT1-UTC, SLR for polar motion and LOD, VLBI for Y polar motion and UT1-UTC) would all be "automatically" combined.

4. Using normal equation solutions allows the combined normals to be formed first and then different weights to be used on parameters (or constraints if a constraint model is being used) if necessary. The addition of new observations or deletion of old ones is also quite easy without recreating all the normals. This property would be useful for handling, e.g., observations which become available at the last minute, or observations found to be bad for one reason or another. The solution of the normal equations will still be needed every time, as well as the usual determination of a variance-covariance matrix, but in practice this is not always done entirely or even needed. However, as we have shown in the last section, the solution of the equations can be done very efficiently in comparison to setting them up.

5. One of the most important advantages of doing normal equation combination or grand solutions, perhaps even more so than the high accuracy shown in this study for their ERP solutions, is their ability to easily unify reference frames when solving for station positions. Provided that a sufficient number of colocated stations exist, the normal equation combination or grand solutions automatically provide a single TRS and a single CRS for all systems which have data included. This means that by default, the biases between the currently existing TRS's and CRS's of each system are eliminated (assuming that most station coordinates are solved for), thus establishing what could then be the new CTRS and CCRS.

6.3 Conclusions

Looking back at some of the original questions asked in this study (in section 1.2), we now attempt to provide some basic conclusions beyond those given in section 6.1.

For example, what relative improvement has been seen for the solutions with the effect of all of the data included in them (the normal equation combination and grand solutions) over the individual system or weighted mean solutions? For the experiments performed here, it is clear that the grand solution and to a slightly lessor extent, the normal equation combination solution usually provide the best relative results. This is certainly true of the RMS difference, maximum difference, average standard deviation for UT1-UTC, and standard deviation results. It can also be seen that the correlations tend to be negligible or lower than in the other methods.

However, the improvement over the weighted mean (and in some cases the individual system) results **is not great.** The RMS and maximum difference results for the weighted mean solutions tend to occasionally be the best or at worst, one to three times larger than the best method. The overall standard deviations (Table 27) are even better than the normal equation combination values, and only just slightly worse than the grand solution values.

What can we say about the individual systems? Only that when specific statistics or results are looked at, do any of them really stand out in comparison to the combination solutions. For example:

1. VLBI often gives the highest or nearly the highest accuracy Y polar motion values (although never the best standard deviations). It also provides UT1-UTC usually within a factor of one to three of the best method.

2. LLR gives the highest accuracy values for UT1-UTC for one- and two-day ERP.

3. Both SLR and VLBI provide polar motion generally at most one to three times worse than the best method.

4. SLR is the only method which provides polar motion parameters with negligible correlations between them.

However, except for some low values for SLR polar motion and VLBI UT1-UTC, the average standard deviations for the individual systems were all substantially higher (more than three times higher) than that of the best methods.

The average difference (bias) results tend to be more ambiguous than the other statistics. The combination solutions tend to provide the lowest values, except for in UT1-UTC. The large UT1-UTC biases in SLR for long-period ERP seem to degrade the normal equation combination and grand solutions, **but not** the weighted mean solutions.

In summary, it would appear that the real conclusions here are dependent on the question: "How much relative improvement over old methods is needed before a new method can be considered?" If a factor of between one and two, and at times three, is not considered a significant improvement, then the use of the normal equation combination and grand solutions should not be considered further, i.e., this implies that **unless the other advantages of the normal equation combination or grand solutions** (just given in the last section) **were felt particularly important,** the **weighted mean** solution alone may be adequate for ERP determination.

Additionally, it appears that the current four-station IRIS VLBI network, operating continuously, might provide ERP of nearly the highest (or the highest for Y polar motion) possible accuracy, except with precision mostly two or more times worse than with the combination solutions. The addition of one station to improve the geometry might even allow it to achieve such high accuracy in X polar motion as well. Except for possibly verifying whether systematic error exists in the VLBI solutions, or to obtain long-period UT1-UTC from LLR, this means that laser ranging may no longer be needed for ERP determinations. This particular conclusion must be considered very tentative however, since as was pointed out earlier, the simulations done here were not designed to compare the individual observing systems; the models and treatment of systematic errors are not complete, and operational problems were not considered. However, in some respects the laser systems have been favored by this assumption, since complete orbit models for the Moon and Lageos could result in larger systematic errors in the ERP than would be expected from using complete VLBI models.

A further conclusion from the simulations is that a "plateau" of sorts has been reached in terms of ERP accuracy, with the highest achievable current accuracies being about 1 mas for polar motion and 0.2 to 1.0 ms for UT1-UTC, as the values of Table 28 indicate. Since we have already assumed the highest possible data rates and the lack of any systematic errors, higher accuracies can only be achieved by more accurate instruments, more instruments, better orbit determination procedures, or radically new methods of ERP determination.

6.4 Further Work

A review of the above conclusions in particular and the other results summarized in this chapter in general, suggests a few possibilities for future work. This is especially true if it is felt that the slight improvements in the ERP results provided by the normal equation combination or grand solutions or their other advantages listed above justify further research. Specific suggestions are:

1. The simulation experiments could be repeated with other observing schedules to see how the ERP are recovered by the various methods during periods of sparse data.

2. The time length of the simulations might be extended to see what problems might occur when considering the long-term stability of the Moon and Lageos's orbits.

3. It has been assumed that slight changes in weighting for the orbit, radio source positions, station positions, etc. parameters, would have no effect on the ERP results. This might be investigated further.

4. To study the effects of (modeled and unmodeled) "systematic error," the simulations should also be repeated using complete models. Adding biases to the observations (e.g., for tropospheric refraction) and seeing how the ERP are recovered when such unmodeled biases exist would also be a worthwhile study.

5. It is especially important to investigate whether the addition of one suitable VLBI station would substantially improve the VLBI X polar motion determinations. If so, this would further indicate that the use of VLBI alone for ERP determination may be possible.

6. Simulations similar to the ones done here could be done to see how well the data combination solution can recover reference frame biases.

7. It is obvious that the experiments in this study could be carried out with real data, although this is not too strongly recommended since the true ERP would then no longer be available as a standard of comparison. In addition, software would still need to be developed further to allow LLR data processing.

8. The use of other types of data to determine ERP could also be compared, by doing simulations to determine absolute possible accuracies.

9. Finally, in the late stages of this research it has come to the author's attention that it may be possible to "iterate" the normal equation combination solution without reforming the normal equations completely (as was done here for the grand solutions). This could be done, albeit not rigorously, by correcting the constant vector of the normal equations for the changes in the parameters from their initial approximate values. The normal coefficient matrix would remain unchanged, and thus not rigorously correct, but assuming a nearly linear solution, it could still be used with the converted constant vector to compute new parameter values, a smaller a posteriori variance of unit weight, and other adjustment results [Estes, 1983, pp. 2-18 to 2-18.1]. This method would provide substantial savings in computer time over the grand solution method. However, research would be needed to see if differences in the results between this and the grand solution are negligible.

Obviously, much more research could be done in the area of combination solutions to obtain Earth Rotation Parameters. It is fairly clear from this study that such solutions are capable of slightly increased accuracy over solutions from individual observing systems, but perhaps not significantly so. Research into the other possible advantages of these solutions may be the most important area for future work, or of course into the obvious alternative, that of improving the individual system results.

APPENDIX

SOLUTION VARIANCES OF UNIT WEIGHT

Table 29 presents the solution a posteriori variance of unit weight for each of the solutions (except of course the weighted mean and mean solutions) described in detail in Chapter 5.

Table 29 Solution A Posteriori Variances of Unit Weight

Recovery Period	LLR	SLR	VLBI	Normal Eq.	Grand Sol.
6 hours	0.57	0.54	1.02	2.40	0.62
6 hours, no LLR		0.54	1.02	2.06	0.63
12 hours	5.10	5.59	58.4	21.1	18.8
1 day	8.83	9.79	131.	36.8	33.9
2 days	8.86	10.0	127.	35.7	33.1
5 days	10.9	11.7	143.	47.2	39.6

Notes:

1. "normal eq." is the normal equation combination solution.
2. The "6 hours, no LLR" indicates that no LLR data was used in the normal eq. and grand solutions.

Three comments need to be added concerning the magnitudes of these values. First, since the normal equation combination solution is a "one iteration" solution, it is not fully converged and always has higher values than the similar, but in effect two iteration, grand solution. Second, for the laser solutions, although the noise added onto the simulated observations had RMS's ranging from 2.3 to 14.5 cm, a constant standard deviation of 10 cm was used to weight all of the laser observations. This being on average somewhat pessimistic, variances less than one result for the six-hour ERP recovery solutions, and the laser solutions always tend to have smaller variances than the VLBI solutions. Therefore, to some extent, the VLBI solutions have more weight than they should in the normal equation combination and grand solutions. Last, as explained previously (5.1.6), since the data contain ERP which change every six hours, any attempts to recover ERP with longer periods results in an apparent high noise level, and a high a posteriori variance of unit weight. The high values shown for 12-hour to five-day ERP recovery are a result of this "model" error, and indicate that ERP solution precisions are sensitive to the ERP parametrization, and the much higher variances of the VLBI solutions for 12-hour to five-day recovery are also further explained by indicating a high sensitivity of VLBI to the unmodeled ERP changes in the data.

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